# 电路理论（2）B考试范围

- ✓动态电路的输入—输出方程
- ✓线性一阶电路（三要素法）
- ✓单位阶跃响应、冲激响应
- ✓正弦稳态电路的分析（相量法、功率计算）
- ✓谐振
- ✓理想变压器、耦合电感
- ✓对称三相电路
- ✓谐波分析法

# 第11章 非正弦周期信号线性电路的稳态分析

## 一、非正弦周期电压电流的有效值的计算

$$u(t) = \mathrm{U}_0 + \sum_{k=1}^{\infty} U_{km} \sin(k\omega t + \theta_k)$$

$$\mathrm{U} = \sqrt{\frac{1}{T}\int_0^T u^2(t)}$$

$$\mathrm{U} = \sqrt{U_0^2 + \sum_{k=1}^{\infty} \mathrm{U}_k^2}$$

谐波电压有效值

$$\mathrm{i}(t) = \mathrm{I}_0 + \sum_{k=1}^{\infty} \mathrm{I}_{km} \sin(k\omega t + \theta_k)$$

$$\mathrm{I} = \sqrt{\frac{1}{T}\int_0^T \mathrm{i}^2(t)}$$

$$\mathrm{I} = \sqrt{\mathrm{I}_0^2 + \sum_{k=1}^{\infty} \mathrm{I}_k^2}$$

谐波电流有效值

# 二、非正弦周期电路有功功率的计算

$$\mathrm{i}(t) = \mathrm{I}_0 + \sum_{k=1}^{\infty} \mathrm{I}_{km} \sin(k\omega t + \theta_{\mathrm{i}k})$$

$$u(t) = \mathrm{U}_0 + \sum_{k=1}^{\infty} U_{km} \sin(k\omega t + \theta_{\mathrm{u}k})$$

$$P = \frac{1}{T}\int_0^T u(t) i(t) dt$$

直流分量和同频率的电压电流产生的功率

$$\mathrm{P} = \mathrm{U}_0\mathrm{I}_0 + \sum_{k=1}^{\infty} \mathrm{I}_\mathrm{K}\mathrm{U}_\mathrm{K} \cos(\theta_{\mathrm{u}k} - \theta_{\mathrm{i}k})$$

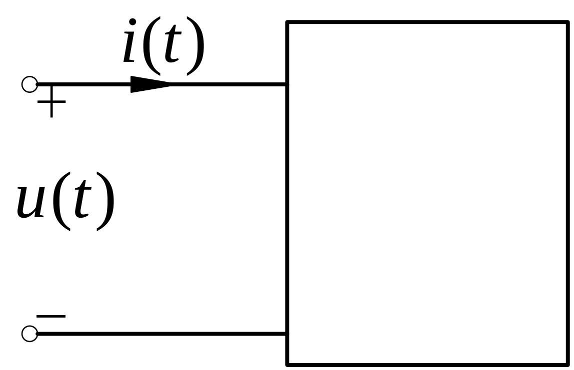
**例：**

$$u(t) = 20 + 5\sqrt{10}\sin(\omega t + 60^0) + 10\sqrt{2}\sin(3\omega t + 75^0)V$$

$$\mathrm{i}(t) = 2 + \sin(3\omega t + 30^0)$$

$$\mathrm{U} = \sqrt{U_0^2 + \sum_{k=1}^{\infty} \mathrm{U}_{\mathrm{k}}^2} = \sqrt{20^2 + \left(5\sqrt{10}/\sqrt{2}\right)^2 + (10\times\sqrt{2}/\sqrt{2})^2}$$

$$= \sqrt{400 + 125 + 100} = 25\mathrm{V}$$



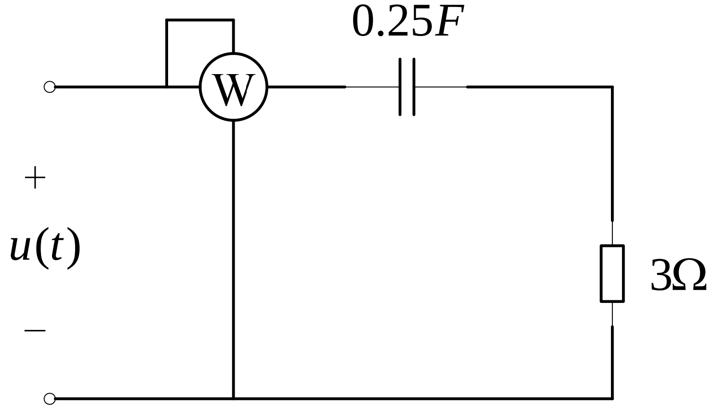

$$\mathrm{I} = \sqrt{2^2 + \left(1/\sqrt{2}\right)^2} = \sqrt{4.5} = 2.12\mathrm{A}$$

$$\mathrm{P} = 20\times 2 + 10\times 1/\sqrt{2}\times\cos(75^0 - 30^0)$$

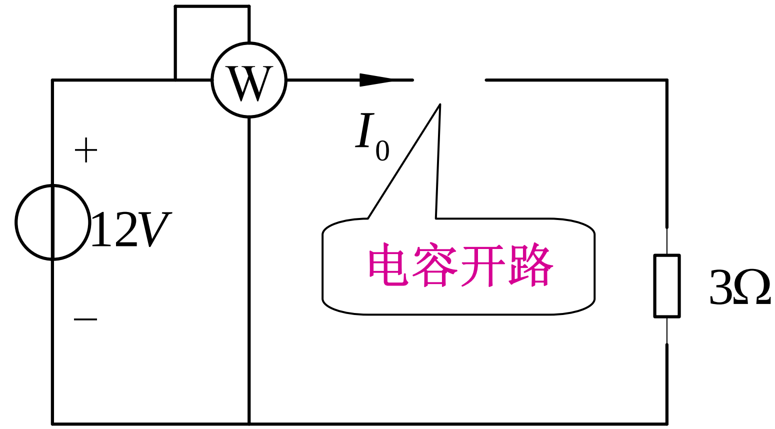
$$=40+5=45\mathrm{W}$$

同频率电压电流

例：$u(t)=12+12\sqrt{2}\sin(t+60^0)V$

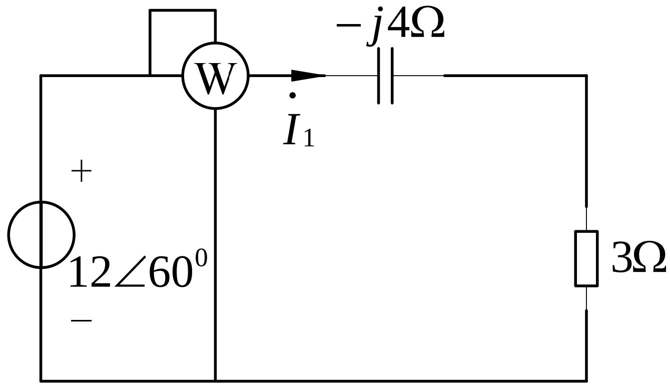



解　1）12V直流电源作用




$$I_0=0A$$

2）交流电源作用（相量模型）




$$\dot{I}_1=\frac{12\angle 60^0}{3-4j}=\frac{12\angle 60^0}{5\angle -53^0}=2.4\angle 113^0$$

$$i_1(t)=2.4\sqrt{2}\sin(t+113^0)$$

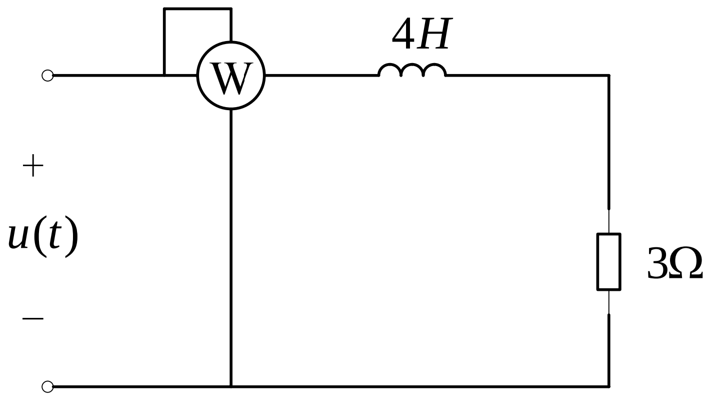
3）时域叠加

$$i(t)=I_0+i_1(t)=2.4\sqrt{2}\sin(t+113^0)$$

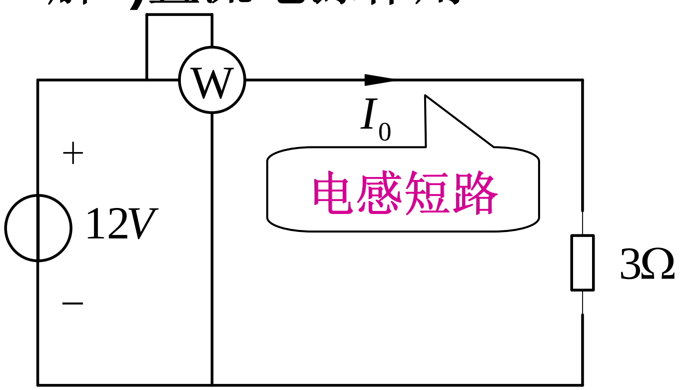
4）功率

$$P=P_0+P_1=12\times 0+12\times 2.4\times\cos\left(-53^0\right)=17$$

**例：** $u(t)=12+15\sqrt{2}\sin(t+60^0)V$



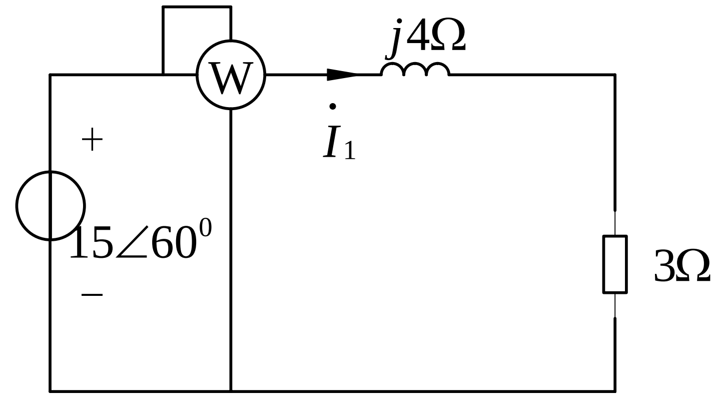

解**1)**直流电源作用




$$I_0=12/3=4A$$

$$P_0=12\times4=48W$$

**2**）交流电源作用（相量模型）




$$\dot{I}=\frac{15\angle60^0}{3+j4}=\frac{15\angle60^0}{5\angle53^0}=3\angle7^0\,A$$

$$i_1(t)=3\sqrt{2}\sin(t+7^0)$$

$$P_1=15\times3\times\cos(53^0)=27W$$

**3**）响应叠加

$$i(t)=I_0+i_1=4+3\sqrt{2}\sin(t+7^0)$$

**4**）功率

$$P=P_0+P_1=48+27=75W$$

# 三、非正弦周期线性电路分析

**叠加定理（时域形式的叠加）**

直流电源和谐波电源作用分别求解

**不同频率下感抗和阻抗的不同**

注意谐振

$$X_C = -\frac{1}{k\omega C} \qquad X_L = k\omega L$$

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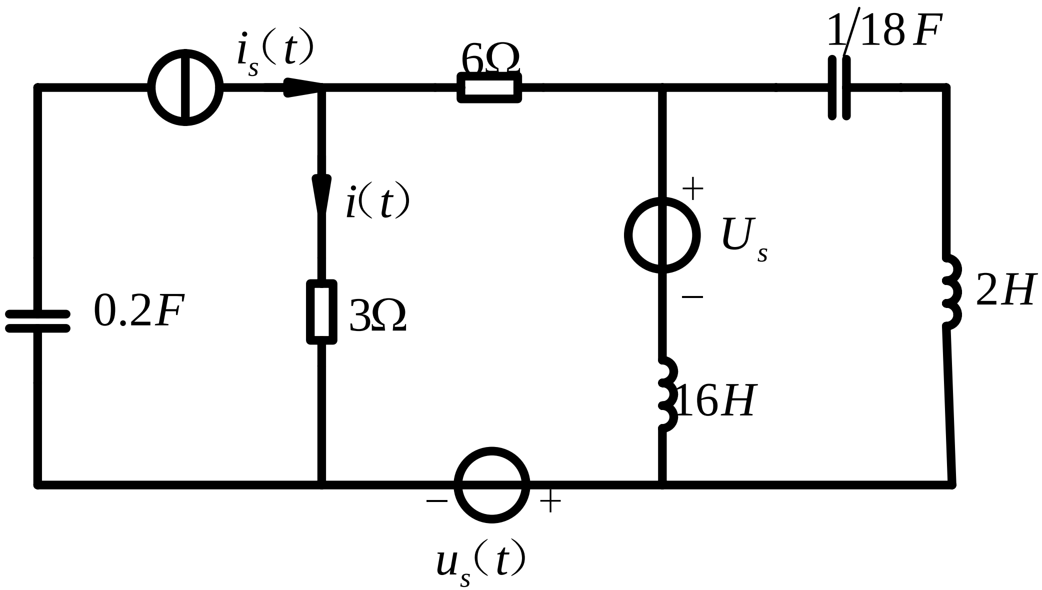
**例1:**



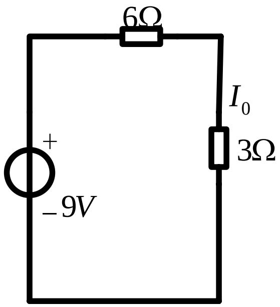

$$u_s(t) = 5\sin(t + 90^0)$$

$$i_s(t) = 3\sqrt{2}\sin(3t + 30^0)$$

$$U_s = 9$$

解：

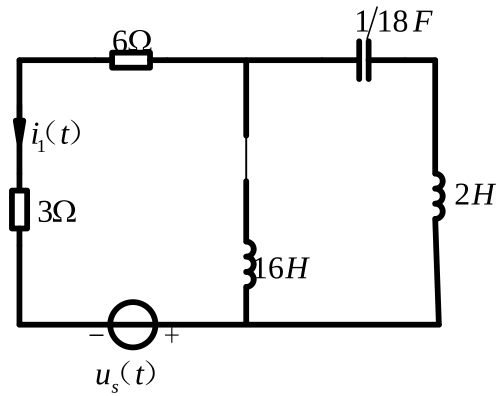
**1）直流电压源作用**




$$I_0 = \frac{9}{3+6} = 1$$

$$P_0 = 3\times 1^2 = 3\text{W}$$

**2）基波电源作用（频率为1rad/s）**

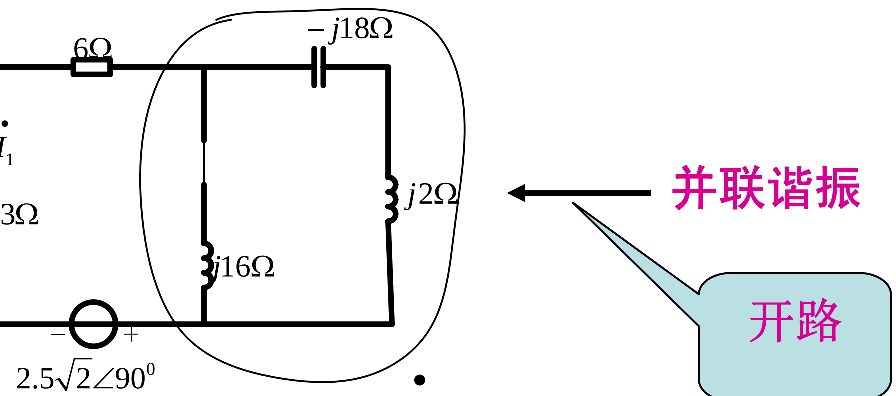



$$X_{L2} = 1\times 16 = 16\Omega$$

$$X_{L1} = 1\times 2 = 2\Omega$$

$$X_C = -\frac{1}{\omega C} = -18$$

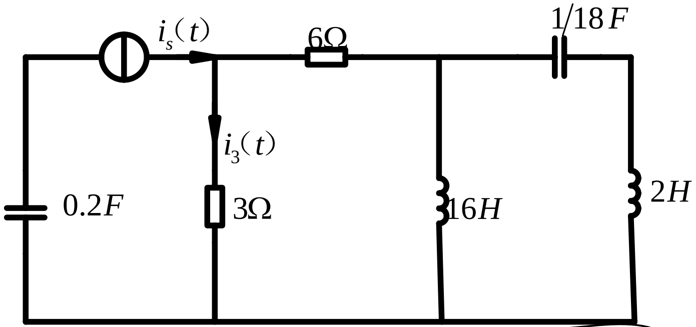
$$\dot{U}_s = 2.5\sqrt{2}\angle 90^0$$




$$\dot{I}_1 = 0$$

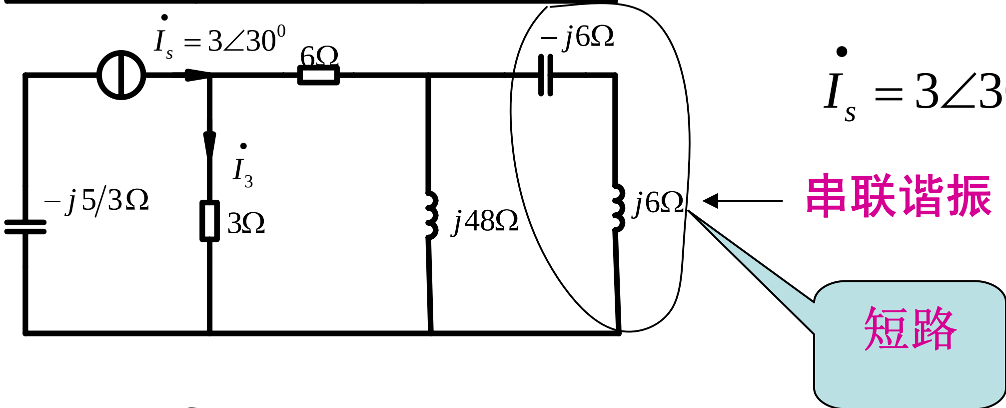
$$P_1 = 3\times 0^2 = 0\text{W}$$

## 3）3次谐波源作用




$$X_{C1}=-\frac{1}{3C}=-6\Omega \qquad X_{C2}=-\frac{5}{3}\Omega$$

$$\boxed{X_{L2}=3\times16=48\Omega} \quad \boxed{X_{L1}=3\times2=6\Omega}$$




$$\dot{I}_s=3\angle30^0$$

$$\dot{I}_3=\frac{6}{(3+6)}\times3\angle30^0=2\angle30^0 \quad i_3(t)=2\sqrt{2}\sin(3t+30^0) \quad P_3=3I_3^{\,2}=3\times2^2=12W$$

4）将结果时域进行叠加

$$i(t)=I_0+i_1(t)+i_3(t)=1+2\sqrt{2}\sin(3t+30^0)A$$

$$I=\sqrt{I_0^{\,2}+I_1^{\,2}+I_3^{\,2}}=\sqrt{1+4}=\sqrt{5}A$$

$$P=3+12=15W$$

# 第10章 三相电路

## 一、三相对称电源

$$\dot{U}_{AN} = U_{ph}\angle 0^0 \qquad \dot{U}_{BN} = U_{ph}\angle -120^0 \qquad \dot{U}_{CN} = U_{ph}\angle 120^0$$

## 二、电源和负载的接法

星形接线和三角形接线方式及线相电压电流的定义

## 三、对称的概念

幅值相同，频率相同，相位差120度

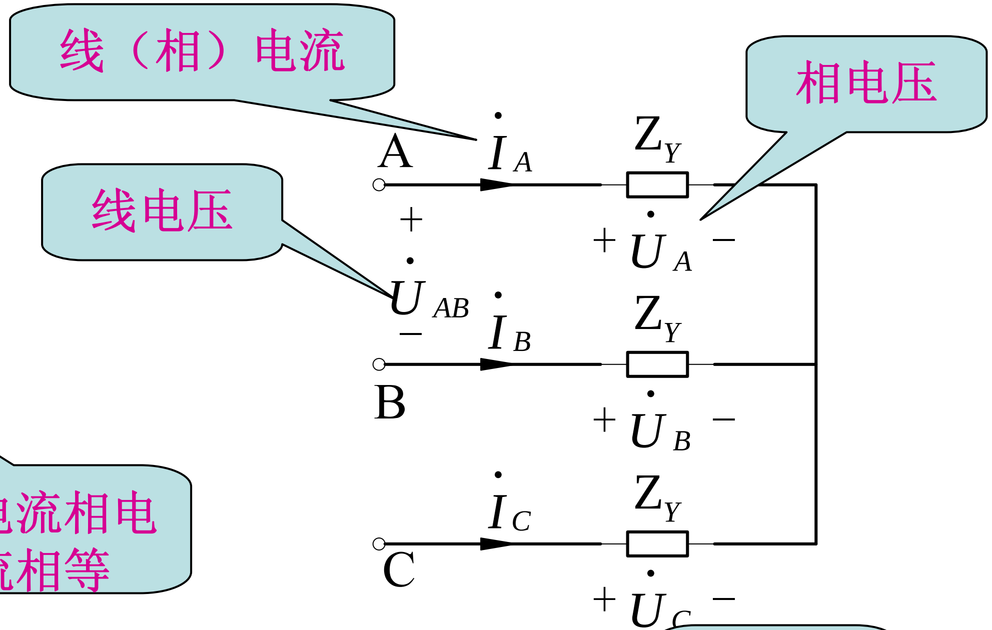
# 四、线相电压线相电流关系（三相对称）

## 星型接法

$$\dot U_{AB}=\sqrt{3}\dot U_A\angle 30^0 \qquad \dot I_A=\frac{\dot U_A}{Z_Y}$$

$$U_l=\sqrt{3}U_{ph} \qquad I_l=I_{ph}$$

线电流相电流相等

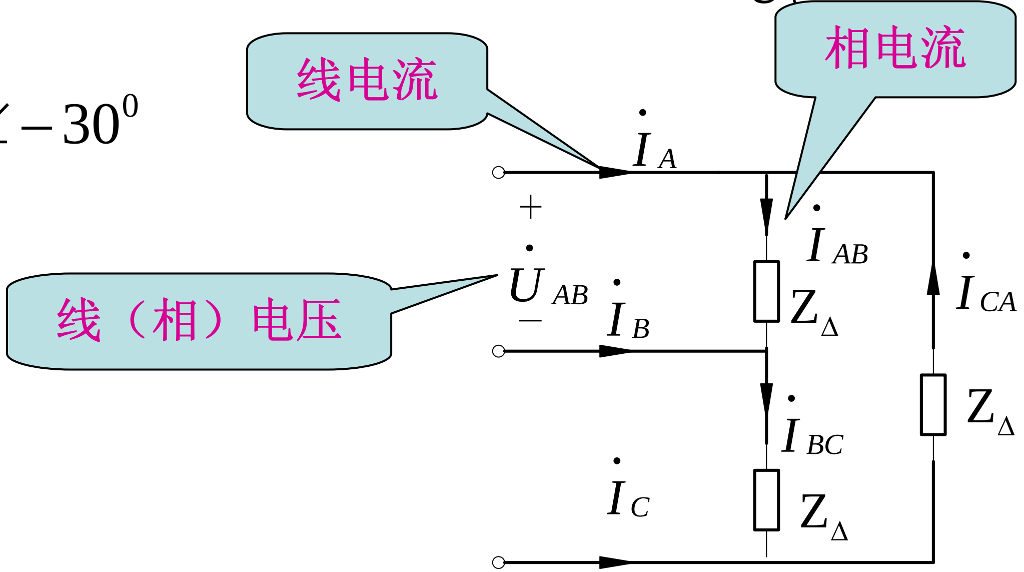



## 三角型接法

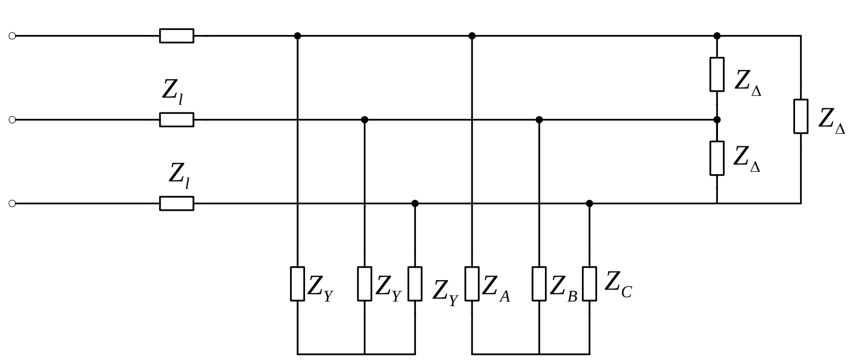
$$\dot I_{AB}=\frac{\dot U_{AB}}{Z_\Delta} \qquad \dot I_A=\sqrt{3}\dot I_{AB}\angle -30^0$$

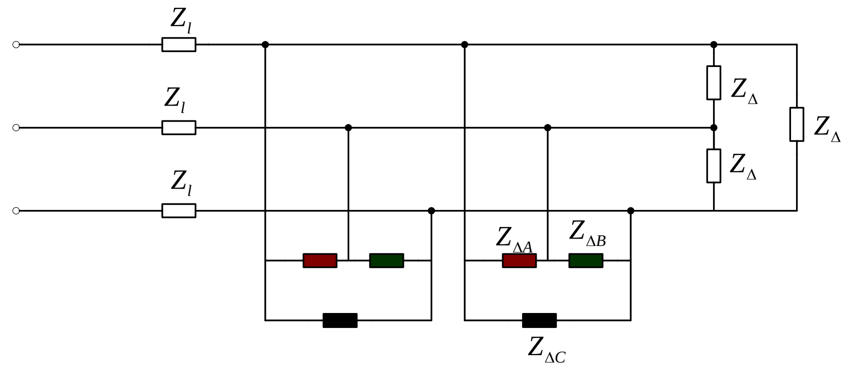
$$U_l=U_{ph} \qquad I_l=\sqrt{3}I_{ph}$$

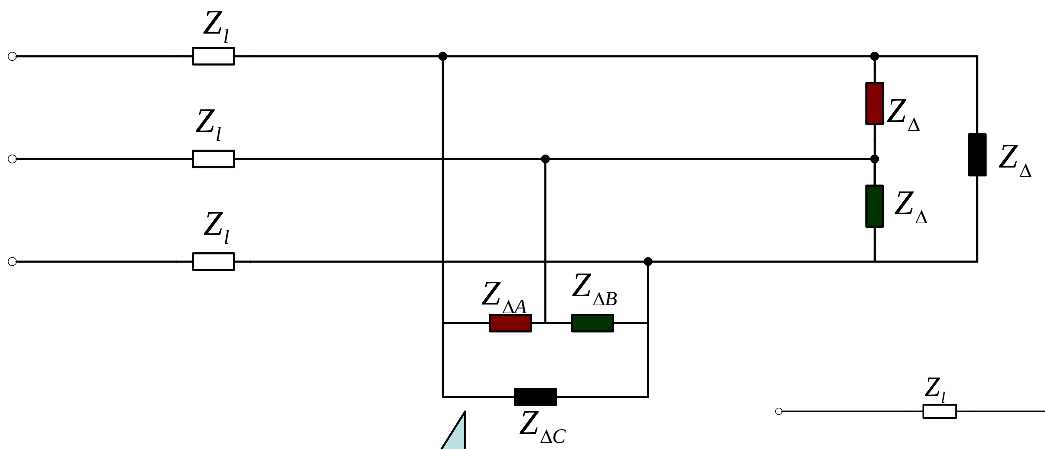
线电压相电压相等

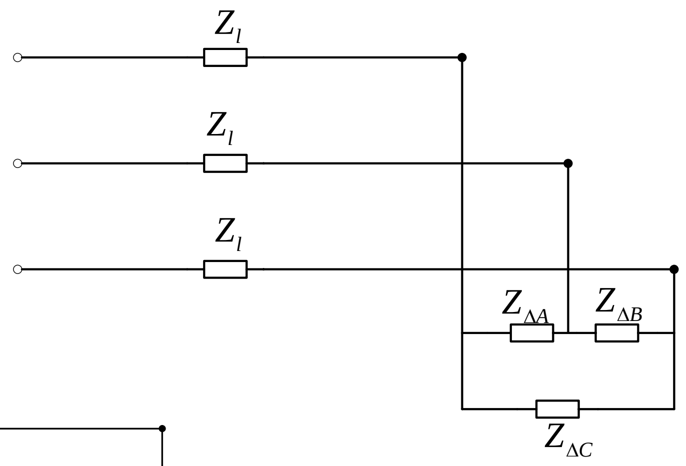


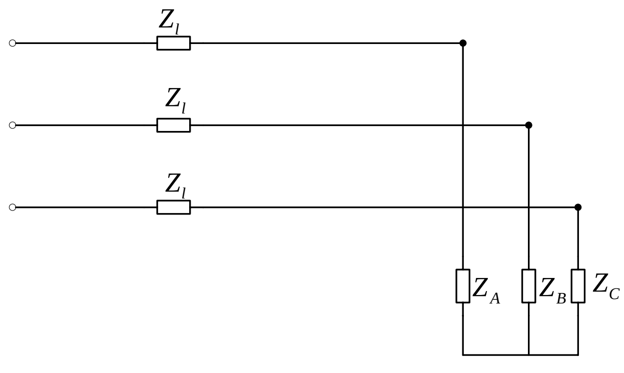
# 五、多相不对称负载



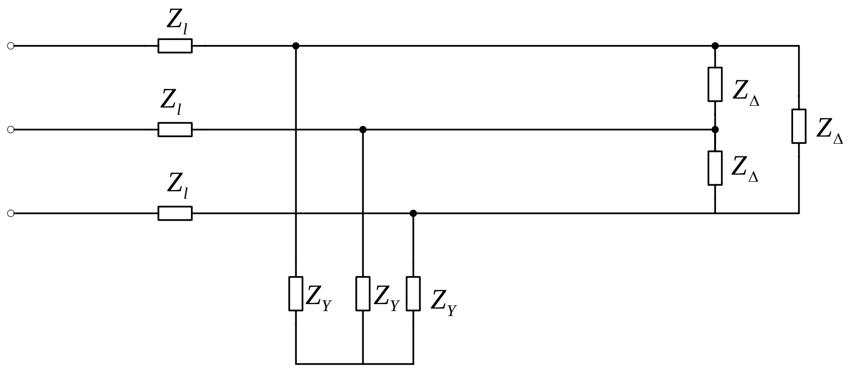



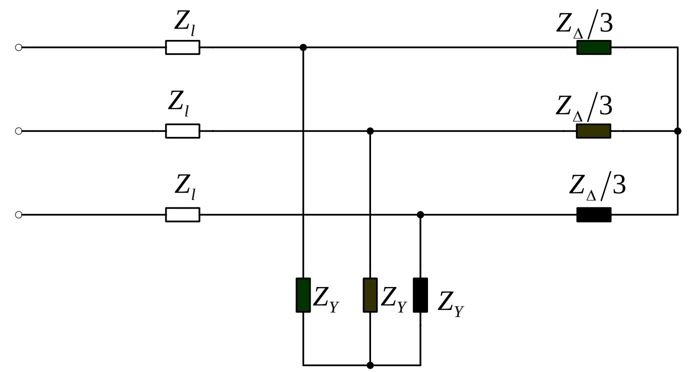




连接于相同相负载并联等效

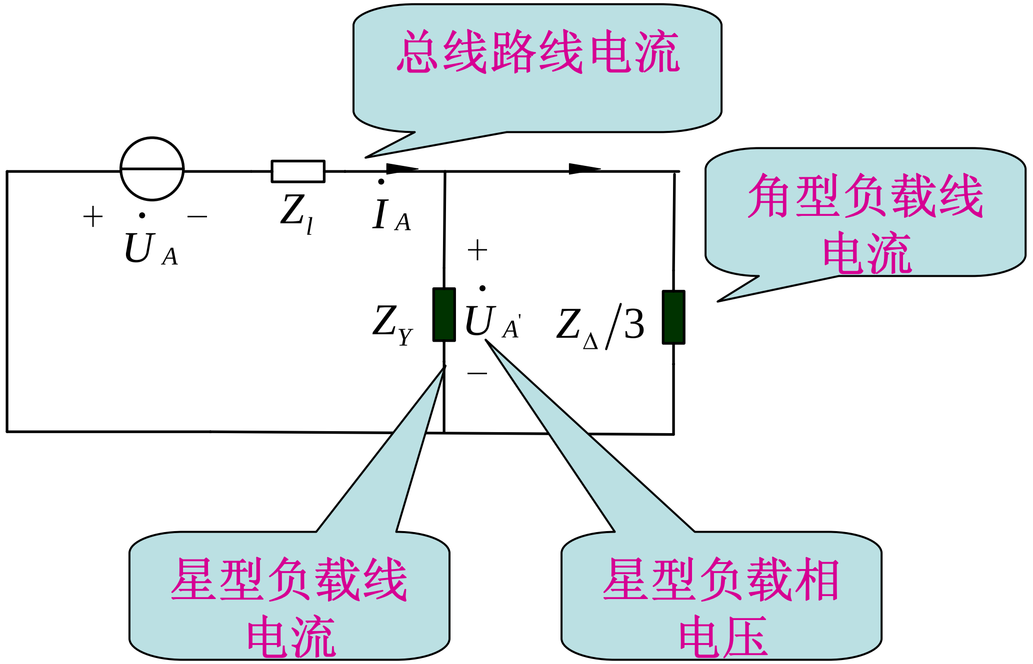
# 六、三相对称电路等效



Z_l
Z_l
Z_l
Z_Δ
Z_Δ
Z_Δ
Z_Y
Z_Y
Z_Y

Z_l
Z_l
Z_l
Z_Δ/3
Z_Δ/3
Z_Δ/3
Z_Y
Z_Y
Z_Y

总线路线电流
角型负载线电流
星型负载线电流
星型负载相电压
+ U_A −
Z_l
I_A
+ U_A' −
Z_Y
Z_Δ/3

# 七、对称三相电路计算

将三相对称电路化为单相电路，根据对称性写出其它相的电压电流

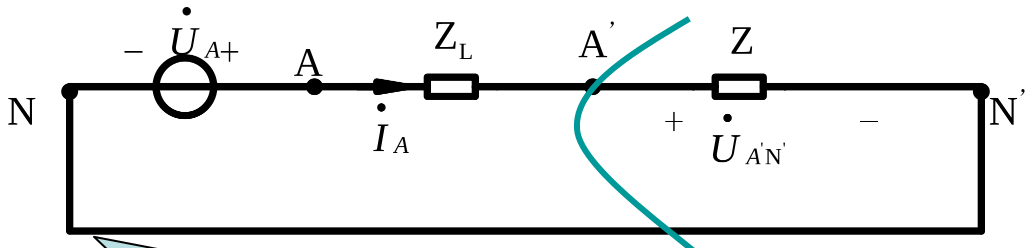
1）根据线相电压关系得出星接电源或负载相电压

2）选参考正弦量

3）根据星三角变换将三角负载化为星型负载(所有的中性点为等电位点)

$$Z_Y = \frac{Z_\Delta}{3}$$

4）根据单相电路计算出电流和电压，根据线相电压电流关系求解出待求量




$$\dot{I}_A = \frac{\dot{U}_{AN}}{Z_l + Z}$$

求得为线电流和相电压

负载与电源中性点等电位

负载侧单相等效电路

# 八、对称三相电路功率计算

## 根据相电压和相电流计算

$$P=3U_{ph}\times I_{ph}\times\cos\theta \qquad Q=3U_{ph}\times I_{ph}\times\sin\theta \qquad I_{ph}=\frac{U_{ph}}{|Z|}$$

## 根据线电压线电流计算

$$P=\sqrt{3}U_{l}\times I_{l}\times\cos\theta \qquad Q=\sqrt{3}U_{l}\times I_{l}\times\sin\theta$$

## 两表法测量与计算

线电压反向

$$P_1=U_{AC}\times I_A\times\cos\left(\theta-30^0\right)$$ 线电压反向计算公式

线电压未反向

$$P_2=U_{BC}\times I_B\times\cos\left(\theta+30^0\right)$$ 线电压未反向计算公式

$$Q=\sqrt{3}(P_1-P_2)$$

$$P=P_1+P_2$$

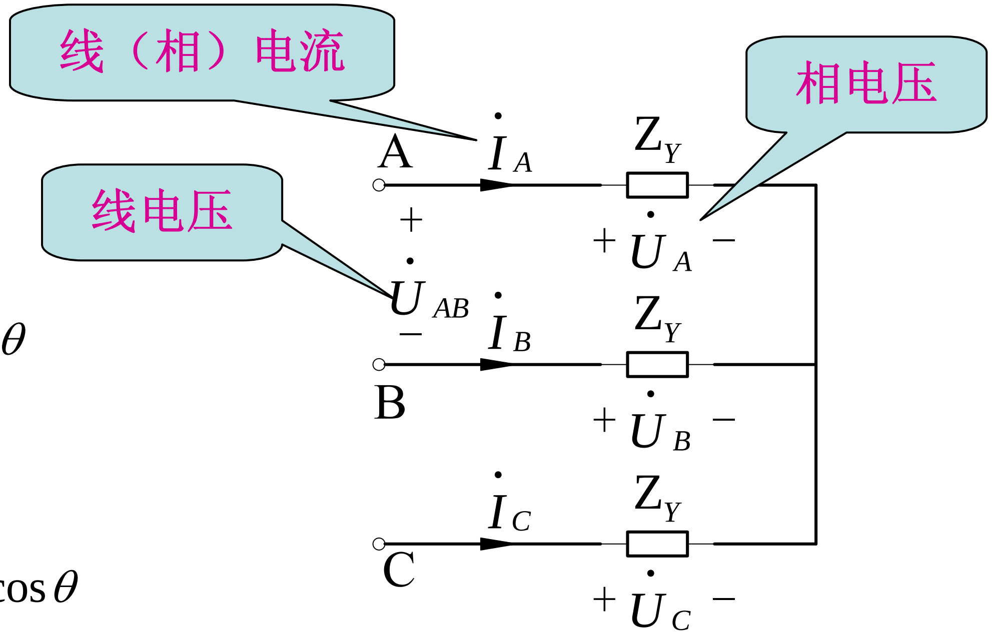
$\theta=\Phi_Z$ 等效负载阻抗的辐角 待计算侧相电压和相电流的相位差

## 功率因数的计算

$$\theta=\theta_{u\_ph}-\theta_{i\_ph}$$

$$Z_{eq}=R+jX \qquad \cos\theta=\frac{R}{\sqrt{R^2+X^2}} \qquad \sin\theta=\frac{X}{\sqrt{R^2+X^2}}$$

## 星型接法




按照相电压相电流计算

$$P = 3U_{Ph} \times I_{Ph} \times \cos\theta = 3U_A \times I_A \times \cos\theta$$

按照线电压线电流计算

$$P = \sqrt{3}U_L \times I_L \times \cos\theta = \sqrt{3}U_{AB} \times I_A \times \cos\theta$$

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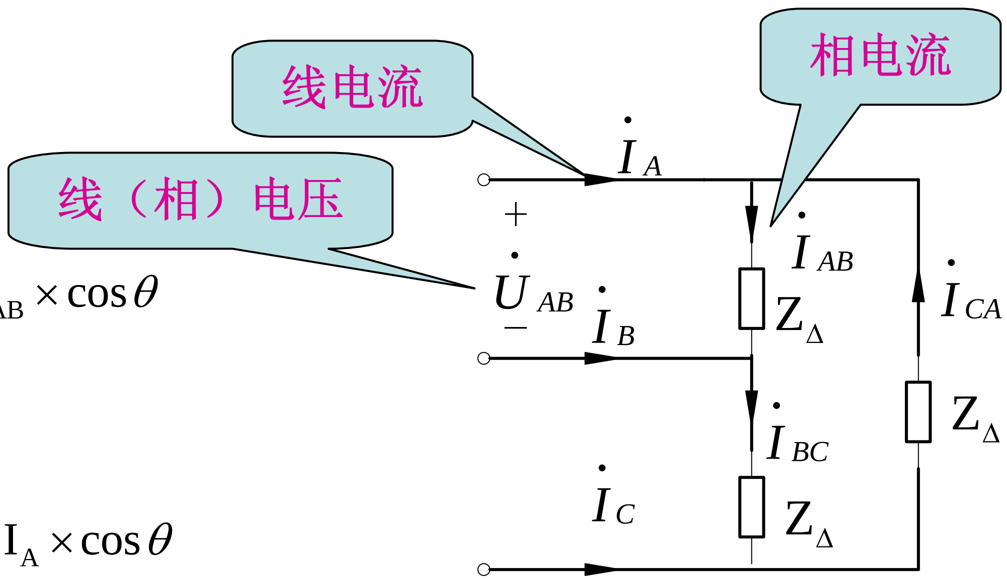
## 三角型接法

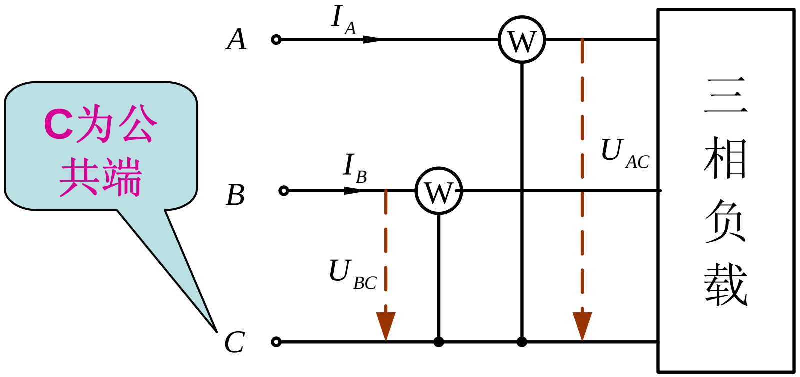



按照相电压相电流计算

$$P = 3U_{Ph} \times I_{Ph} \times \cos\theta = 3U_{AB} \times I_{AB} \times \cos\theta$$

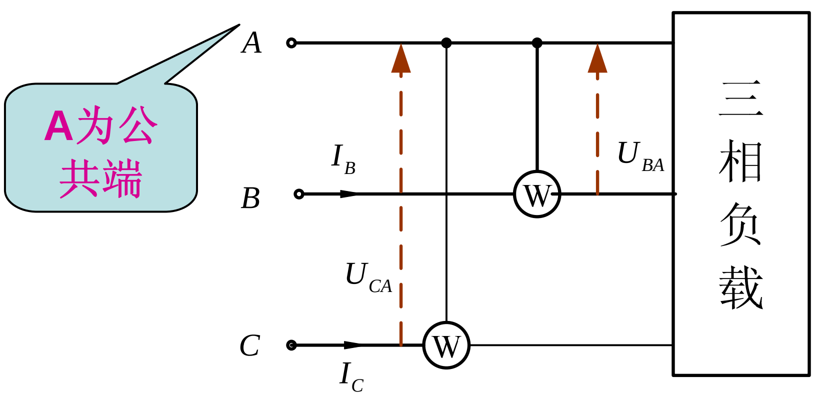
按照线电压线电流计算

$$P = \sqrt{3}U_L \times I_L \times \cos\theta = \sqrt{3}U_{AB} \times I_A \times \cos\theta$$



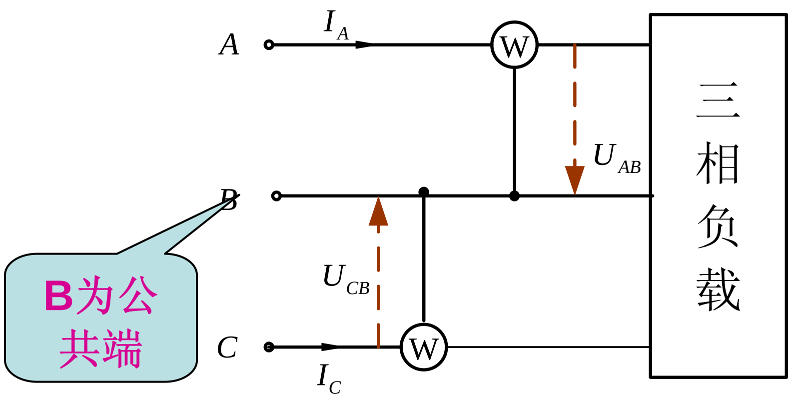

$$\begin{cases} P_1 = U_{AC} \times I_A \times \cos(\theta - 30^0) \\ P_2 = U_{BC} \times I_B \times \cos(\theta + 30^0) \end{cases}$$

线电压反向




$$\begin{cases} P_1 = U_{BA} \times I_B \times \cos(\theta - 30^0) \\ P_2 = U_{CA} \times I_C \times \cos(\theta + 30^0) \end{cases}$$

线电压未反向



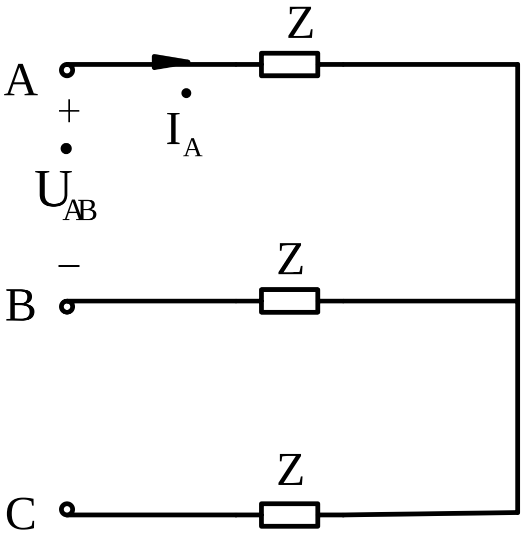

$$\begin{cases} P_1 = U_{CB} \times I_C \times \cos(\theta - 30^0) \\ P_2 = U_{AB} \times I_A \times \cos(\theta + 30^0) \end{cases}$$

$$Q = \sqrt{3}(P_1 - P_2)$$

无功功率

例：三相对称系统参数如下：$U_{AB}=380V \quad Z=8+j6\Omega$

解 **1**）电源为星型接线则，令




$$\dot{U}_A=\frac{380}{\sqrt{3}}\angle 0^0=220\angle 0^0$$

$$\dot{I}_A=\frac{\dot{U}_A}{Z}=\frac{220\angle 0^0}{8+6j}=\frac{220\angle 0^0}{10\angle 37^0}=22\angle -37^0$$

$$\sin\theta=6\Big/\sqrt{8^2+6^2}=0.6 \qquad \cos\theta=8\Big/\sqrt{8^2+6^2}=0.8$$

**2**）

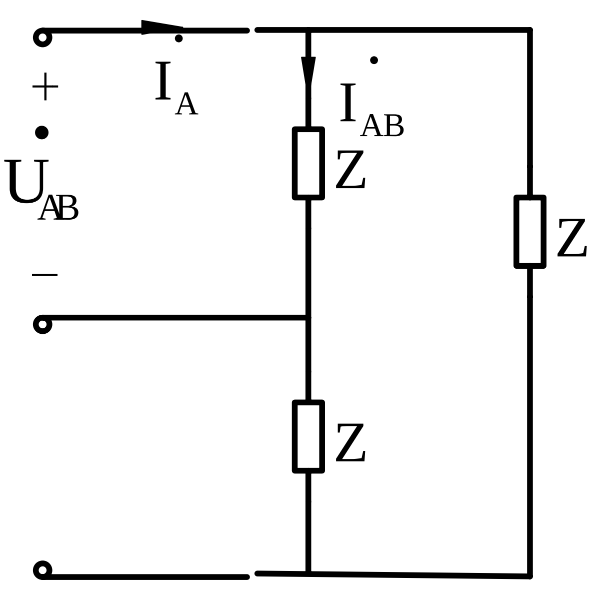
或者 $\theta=0^0-(-)=37^0$

$$P=\sqrt{3}U_l\times I_l\times\cos\theta=\sqrt{3}\times 380\times 22\times 0.8=11616W$$

$$Q=\sqrt{3}U_l\times I_l\times\sin\theta=\sqrt{3}\times 380\times 22\times 0.6=8712\,\mathrm{var}$$

例：三相对称系统参数如下：$U_{AB}=380V$ $Z=8+j6\Omega$

解 1）令 $\dot{U}_{AB}=380\angle 0^0$




$$\dot{I}_{AB}=\frac{\dot{U}_{AB}}{Z}=\frac{380\angle 0^0}{8+6j}=\frac{380\angle 0^0}{10\angle 37^0}=38\angle -37^0$$

$$\dot{I}_A=38\sqrt{3}\angle -37^0-30^0=38\sqrt{3}\angle -67^0$$

2）

$$\sin\theta=6\Big/\sqrt{8^2+6^2}=0.6 \qquad \cos\theta=8\Big/\sqrt{8^2+6^2}=0.8$$

$$P=\sqrt{3}U_l\times I_l\times\cos\theta=\sqrt{3}\times 380\times 38\sqrt{3}\times 0.8=346560\text{W}$$

$$Q=\sqrt{3}U_l\times I_l\times\sin\theta=\sqrt{3}\times 380\times 38\sqrt{3}\times 0.6=722000\,\text{var}$$

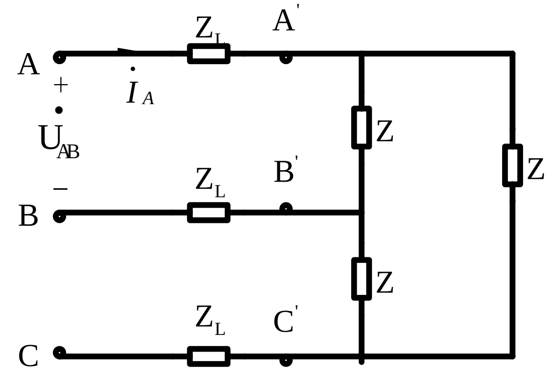
三相对称系统参数如下： $U_{AB}=380V$  $Z_L=5+j2\Omega$  $Z=9+j12\Omega$

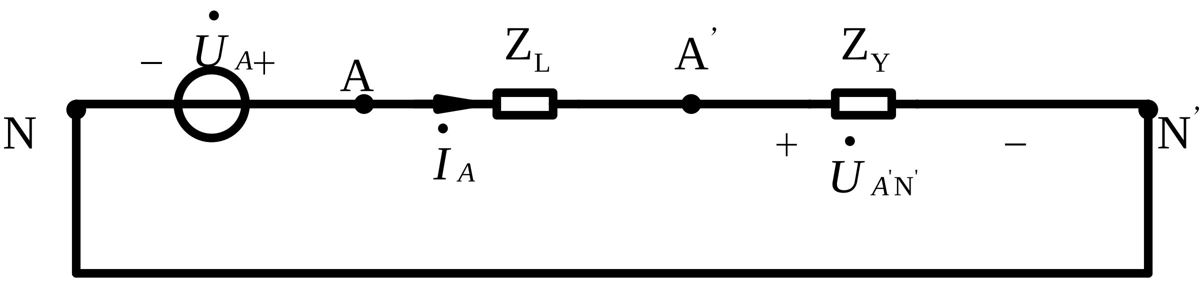
求电路的有功和无功功率

解 **1**）求参数画单相等值电路

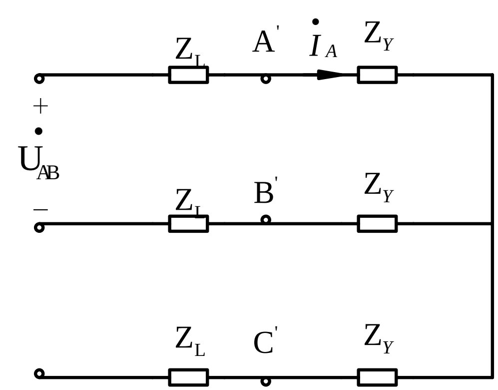
$$Z_Y=\frac{9+12j}{3}=3+j4\Omega$$

$$U_A=380/\sqrt{3}=220$$ 令 $$\dot{U}_A=220\angle 0^0$$



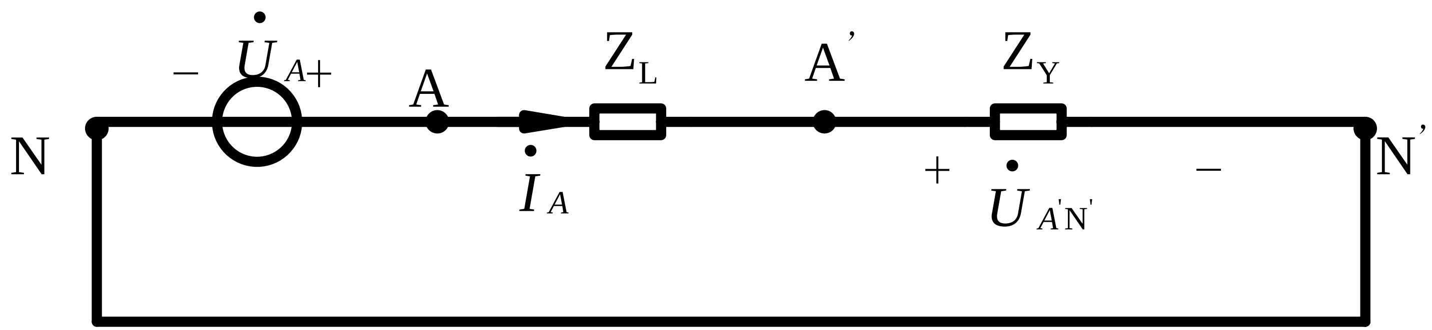




单相等值电路




$$\dot{I}_A=\frac{\dot{U}_A}{Z_L+Z_Y}=\frac{220\angle 0^0}{3+4j+5+2j}$$

$$=\frac{220\angle 0^0}{10\angle 37^0}=22\angle -37^0$$




**2**） $Z = Z_L + Z_Y = 3 + 4j + 5 + 2j = 8 + 6j\Omega$

$\sin\theta = 6 / \sqrt{8^2 + 6^2} = 0.6$　　　　$\cos\theta = 8 / \sqrt{8^2 + 6^2} = 0.8$

或　$\theta = 0 - (-37^0) = 37^0$

待计算电路相电压和相电流的相位差

**3**）

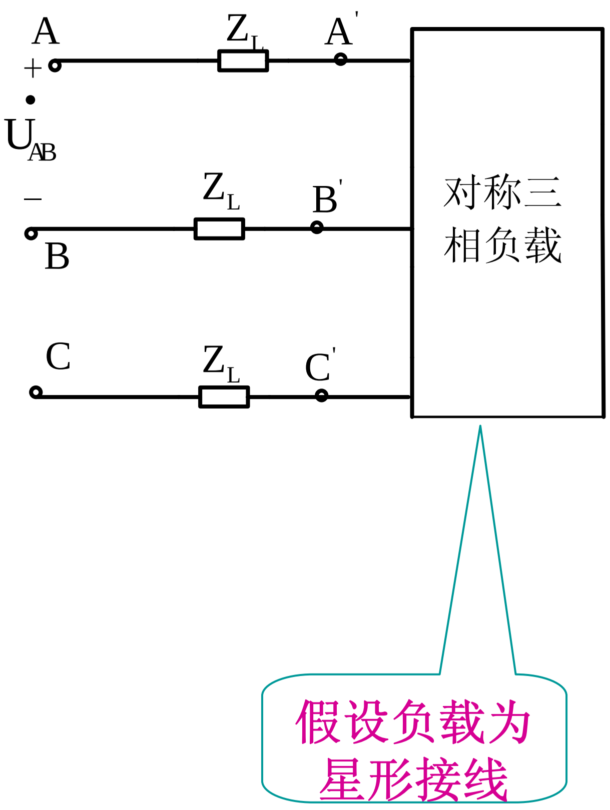
$$P = \sqrt{3}U_l \times I_l \times \cos\theta = \sqrt{3} \times 380 \times 22 \times 0.8 = 11616\text{W}$$

$$Q = \sqrt{3}U_l \times I_l \times \sin\theta = \sqrt{3} \times 380 \times 22 \times 0.6 = 8712\,\text{var}$$

已知负载端线电压为380V，线电流为2A负载功率因数为0.8（感性），线路阻抗为 $Z_l = 4 + j3\Omega$

求电源线电压 $\dot{U}_{AB}$ $\dot{U}_{BC}$ $\dot{U}_{CA}$

求电源提供的有功功率无功功率和视在功率



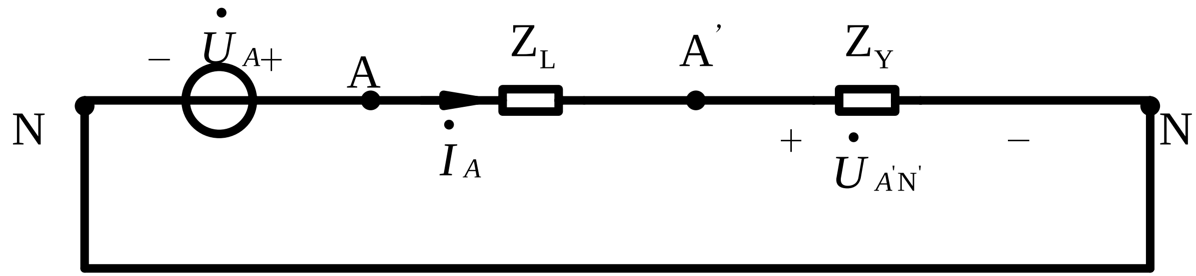

解：**1**）$U_{A'B'} = 380V$ $U_{A'N'} = \dfrac{380}{\sqrt{3}} = 220V$

令 $\dot{U}_{A'N'} = 220\angle 0^0 V$

功率因数为**0.8**（感性）$\theta = 37^0$

则 $\dot{I}_{A'} = \dfrac{\dot{U}_{A'N'}}{Z} = \dfrac{220\angle 0^0}{|Z|\angle 37^0} = 2\angle -37^0$

2）




$$\dot{U}_{AN} = \dot{I}_{A'} \times Z_l + \dot{U}_{A'N'}\ V$$

$$\dot{U}_{AN} = 2\angle -37^0 \times (4+3j) + 220\angle 0^0 = 230\angle 0^0 V$$

$$\dot{U}_{AB} = \sqrt{3}230\angle 30^0 = 398.36\angle 30^0 V$$

3）

$$P = \sqrt{3}U_l \times I_l \times \cos\theta = \sqrt{3} \times 398.36 \times 2 \times 0.8 = 1104W$$

$$Q = \sqrt{3}U_l \times I_l \times \sin\theta = \sqrt{3} \times 398.36 \times 2 \times 0.6 = 828\,\text{var}$$

$$S = \sqrt{3}U_l \times I_l = \sqrt{3} \times 398.36 \times 2 = 1380VA$$

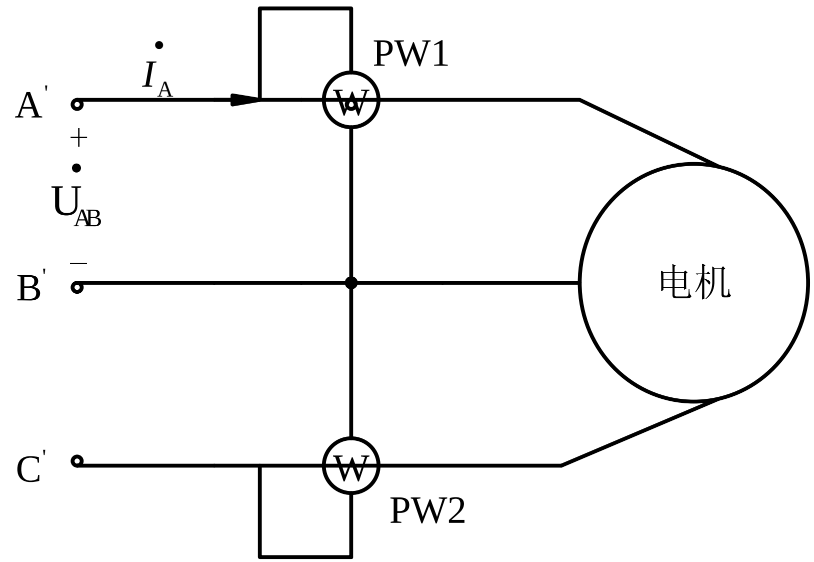
已知电动机功率为2.5kW，功率因数为0.866，对称三相电源线电压为380V，求两个功率表的读数。

解：1）

$$P=\sqrt{3}U_l\times I_l\times\cos\theta$$

$$=\sqrt{3}\times380\times I_l\times0.866=2500W$$

$$I_l=\frac{2500}{\sqrt{3}\times380\times\times0.866}=4.386A$$



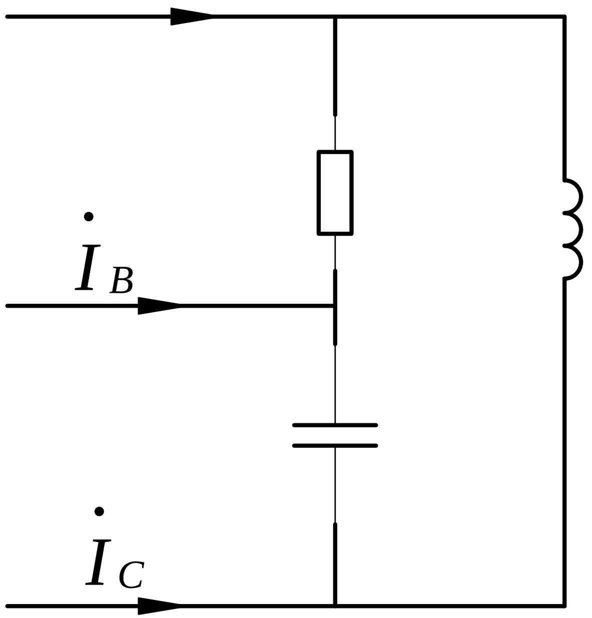

2）$\cos\theta=0.866 \quad \theta=30^0$

$$P_1=U_{AB}\times I_A\times\cos(\theta+30^0)=380\times4.386\times0.5=833.34W$$

$$P_2=U_{CB}\times I_C\times\cos(\theta-30^0)=380\times4.386\times1=1666.68W$$

# 没有线路阻抗的不对称电路

已知各个负载上的电流为**8A**，求**Ic**




解：以**uab**为参考相量

$$\dot{U}_{AB}=U_L\angle 0^0 \quad \dot{I}_{AB}=\frac{U_L\angle 0^0}{R}=8\angle 0^0$$

$$\dot{U}_{BC}=U_L\angle -120^0$$

$$\dot{I}_{BC}=\frac{U_L\angle -120^0}{\frac{1}{\omega C}\angle -90^0}=8\angle -30^0$$

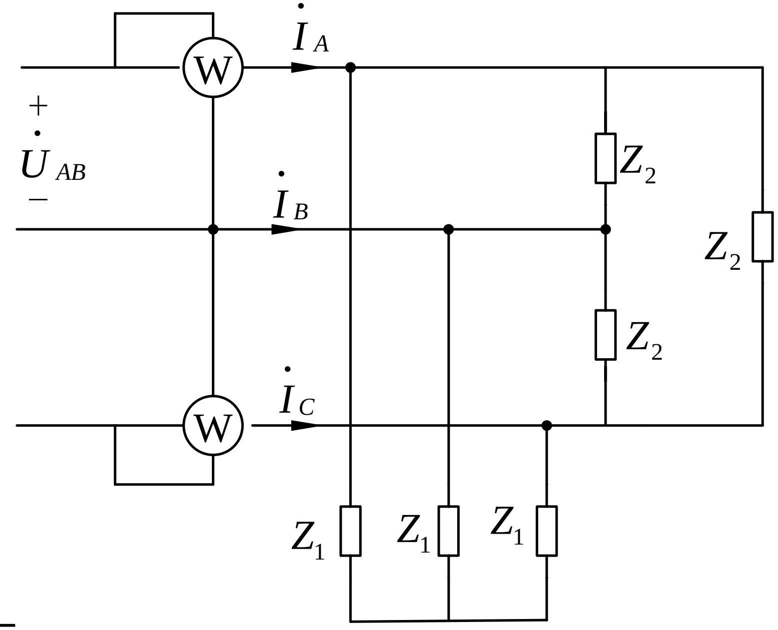
$$\dot{U}_{CA}=U_L\angle 120^0 \qquad \dot{I}_{CA}=\frac{U_L\angle 120^0}{\omega L\angle 90^0}=8\angle 30^0$$

$$\dot{I}_C=\dot{I}_{CA}-\dot{I}_{BC}=8\angle 30^0-8\angle -30^0=8\angle 90^0$$

已知：$\dot{U}_{AB}=173\angle 30^0$

$Z_1=30+40j\Omega \quad Z_2=30\Omega$

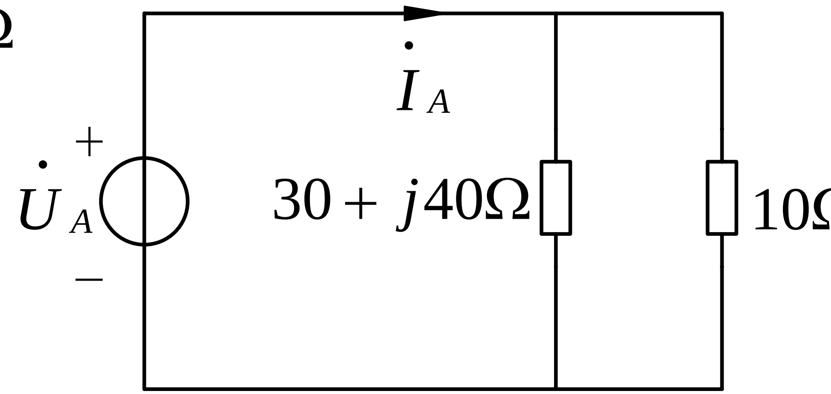
求功率表的示数




解：**1**）求参数画单相等值电路

$Z_Y=\frac{30}{3}=10\Omega \qquad \dot{U}_A=\frac{173}{\sqrt{3}}=100\angle 0^0$

$$Z=\frac{(30+40j)10}{40+j40}=\frac{50\angle 53}{4\sqrt{2}\angle 45}=6.25\sqrt{2}\angle 8\Omega$$




$$\dot{I}_A=\frac{\dot{U}_A}{Z}=\frac{100\angle 0^0}{6.25\sqrt{2}\angle 8^0}=8\sqrt{2}\angle -8^0$$

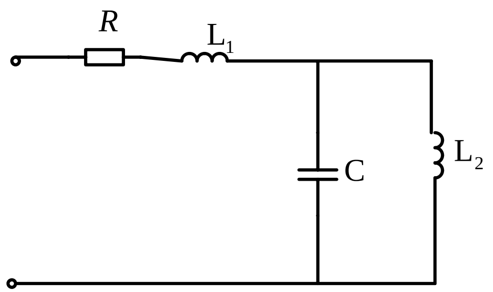
**2**） $\theta=8^0$

$$\mathrm{P}_1=\mathrm{U}_{\mathrm{AB}}\times \mathrm{I}_A\times\cos(\theta+30^0)=173\times 8\sqrt{2}\times 0.8=1565\mathrm{W}$$

$$\mathrm{P}_2=\mathrm{U}_{\mathrm{CB}}\times \mathrm{I}_C\times\cos(\theta-30^0)=173\times 8\sqrt{2}\times 0.927=1814\mathrm{W}$$

# 第9章　谐振与互感

## 一、谐振频率的计算




$$Z = R + j\omega L_1 + \frac{j\omega L_2 \times \frac{1}{jwc}}{j\omega L_2 - j\frac{1}{wc}}$$

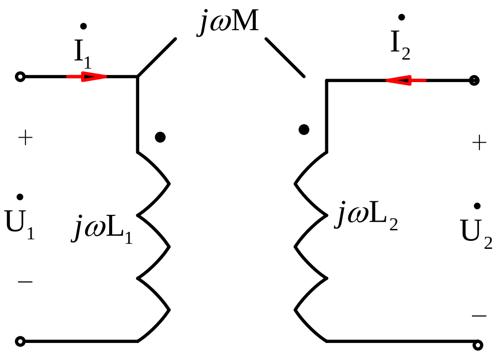
等效阻抗或导纳的虚部为零

$$\omega_1 = \frac{1}{\sqrt{L_2 C}} \qquad \omega_2 = \sqrt{\frac{L_2 + L_1}{L_2 L_1 C}}$$

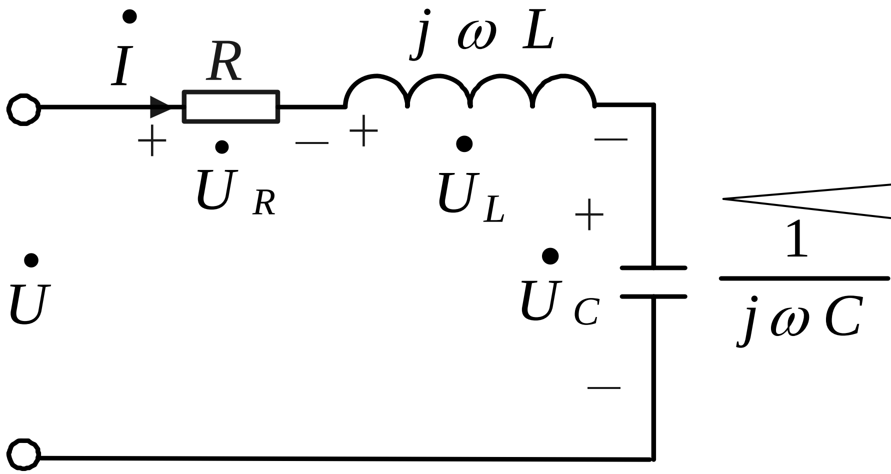
## 二、耦合电感电压电流关系

$$\dot{U}_1 = j\omega L_1 \times \dot{I}_1 + j\omega M \times \dot{I}_2$$

$$\dot{U}_2 = j\omega L_2 \times \dot{I}_2 + j\omega M \times \dot{I}_1$$

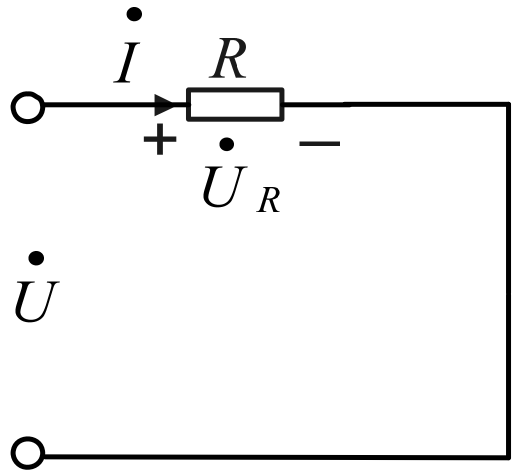



电流流入端对应的同名
端为互感电压的高电位

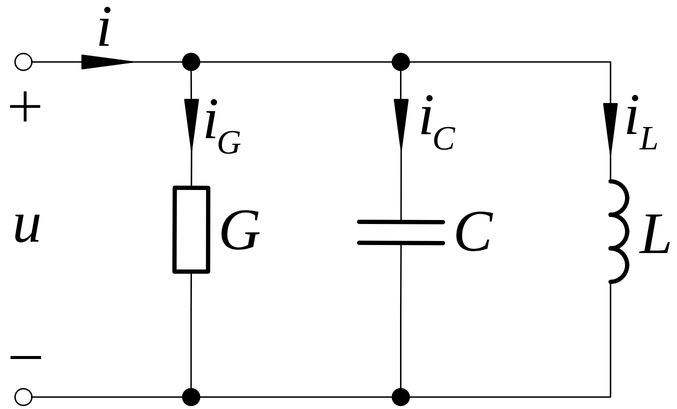


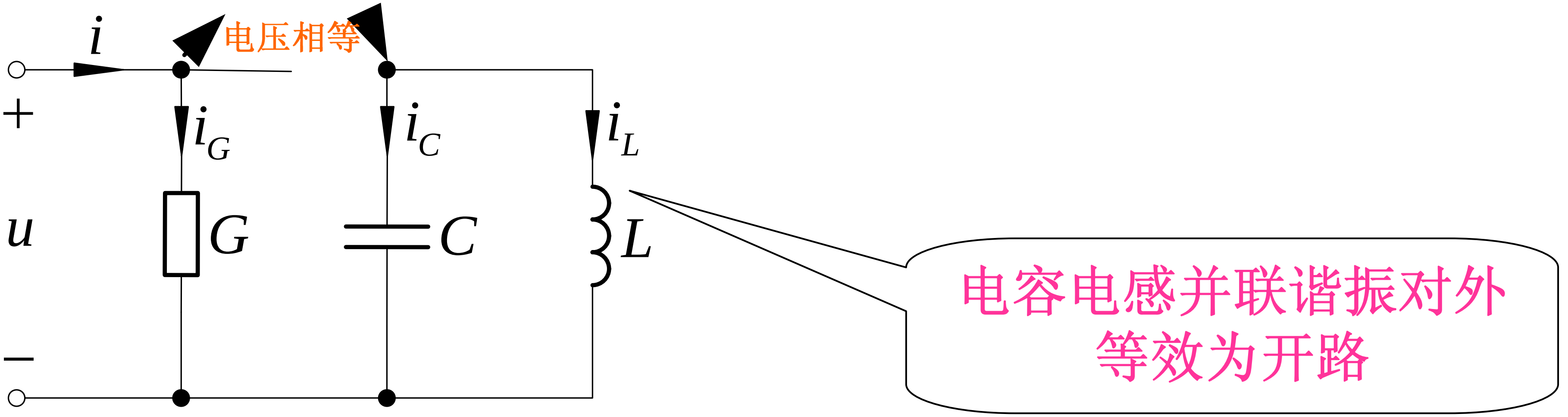

电容电感串联谐振对外等效为短路

**RLC**串联电路



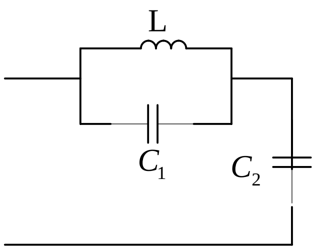

**RLC**串联等效电路



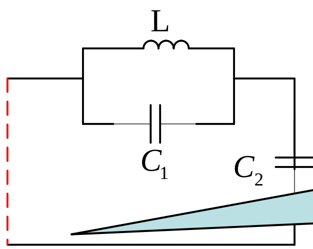




并联等效电路

# 纯电容电感构成谐振电路的谐振频率计算

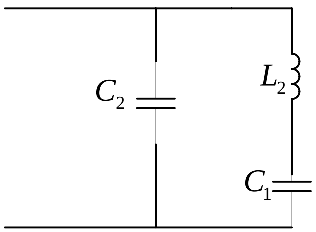



$$\omega_1 = \frac{1}{\sqrt{LC_1}}$$




串联谐振对外等效为短路

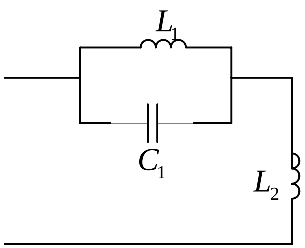
$$\omega_2 = \frac{1}{\sqrt{L(C_1+C_2)}}$$



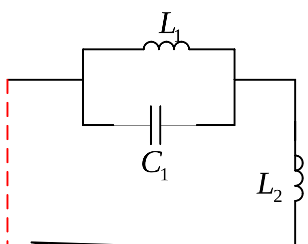

并联谐振对外等效为开路

$$\omega_1 = \frac{1}{\sqrt{L_2C_1}}$$

$$\omega_2 = \frac{1}{\sqrt{L\frac{C_1C_2}{(C_1+C_2)}}}$$

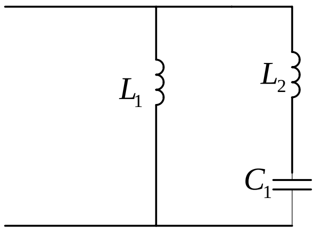



$$\omega_1 = \frac{1}{\sqrt{L_1C_1}}$$




串联谐振对外等效为短路

$$\omega_2 = \frac{1}{\sqrt{C\frac{L_1L_2}{(L_1+L_2)}}}$$




并联谐振对外等效为开路

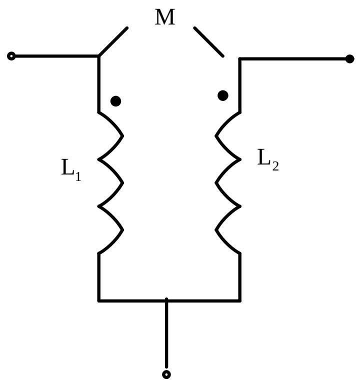
$$\omega_1 = \frac{1}{\sqrt{L_2C_1}}$$

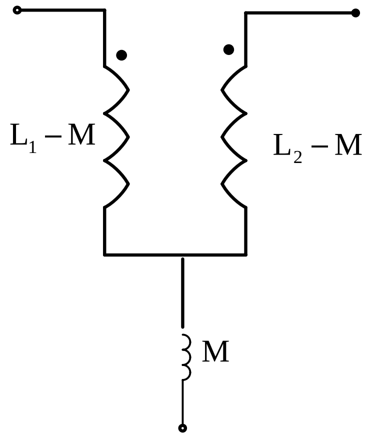
$$\omega_2 = \frac{1}{\sqrt{C_1(L_1+L_2)}}$$

# 三、耦合电感等效电路

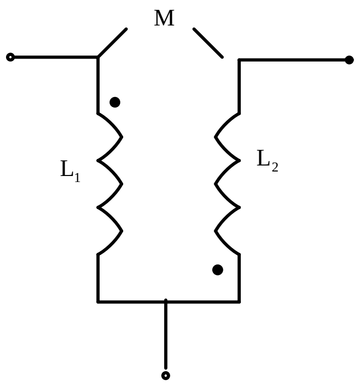
## 去耦合等效

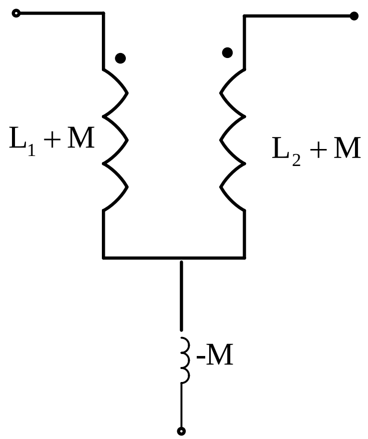
### 同侧并




### 异侧并

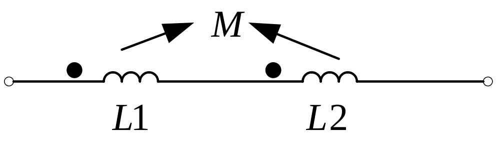


# 四、耦合电感等效电路

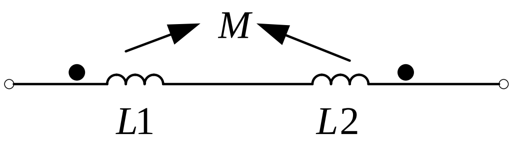
电感串并联等效电感

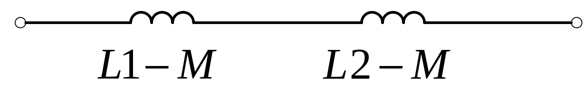
## 去耦合等效



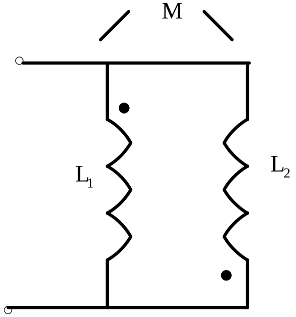


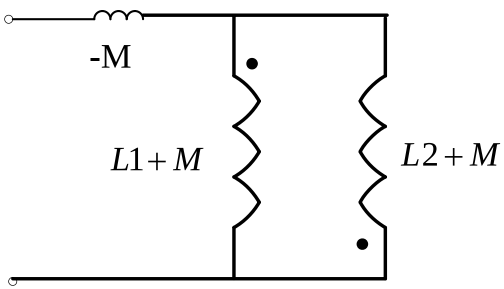
顺接



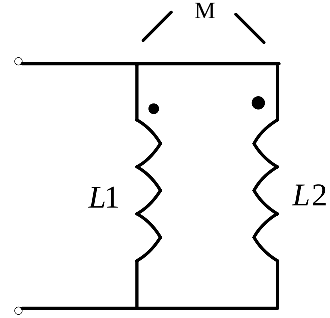



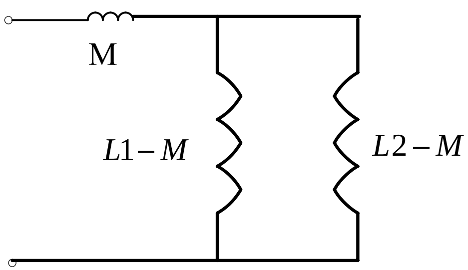

反接




异侧并

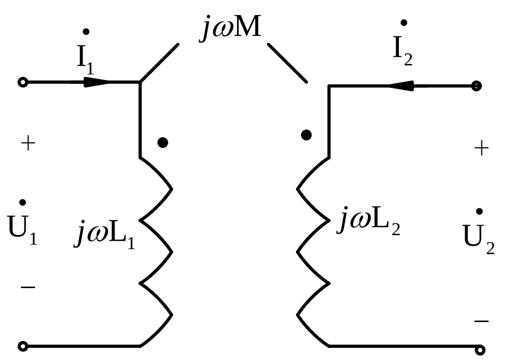


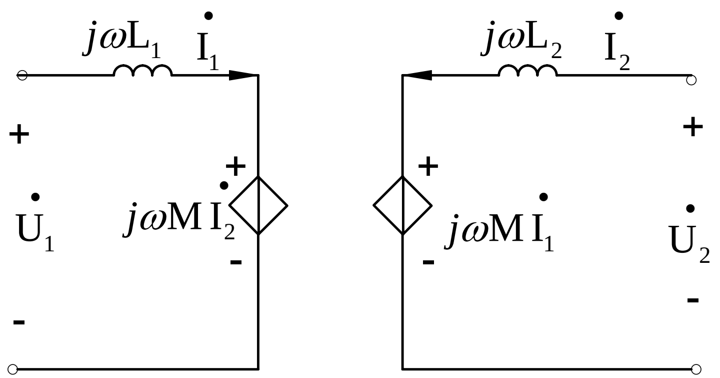




同侧并

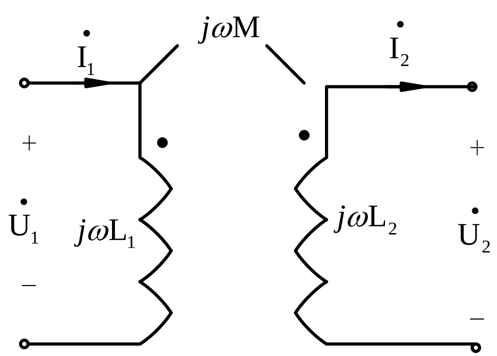
# 五、耦合电感等效电路

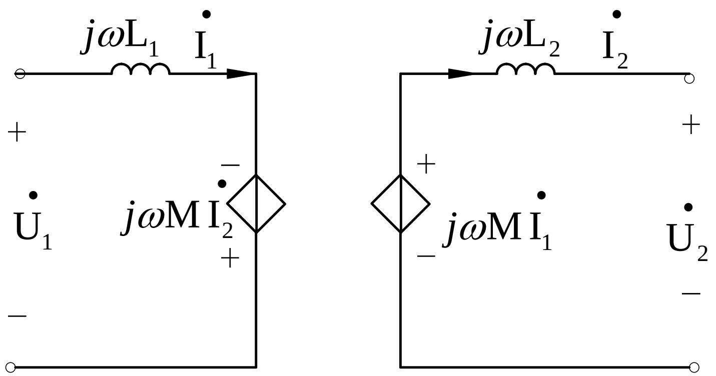
## 受控电压源等效



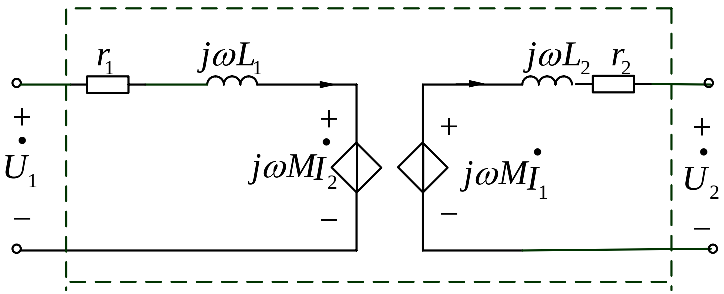



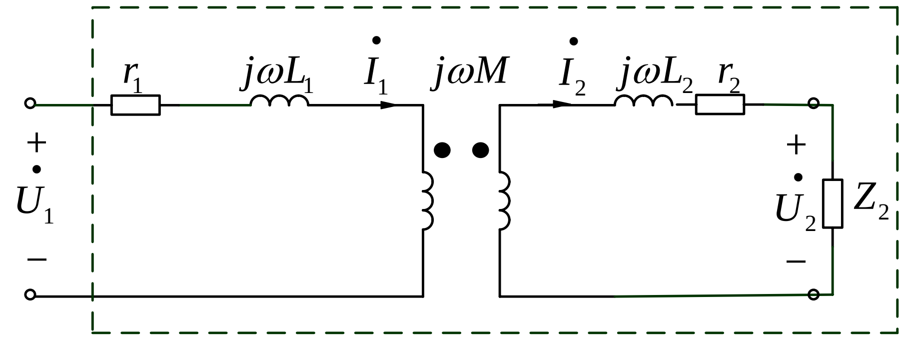

## 受控电压源的参考极性与控制量和同名端相关



# 六、空心变压器（如何识别？）



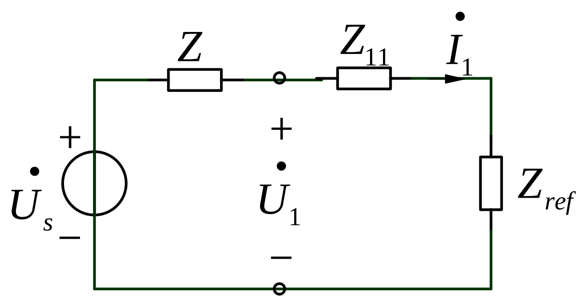

$$Z_{11} = r_1 + j\omega L_1$$

$$Z_{ref} = \frac{(\omega M)^2}{Z_{22}}$$

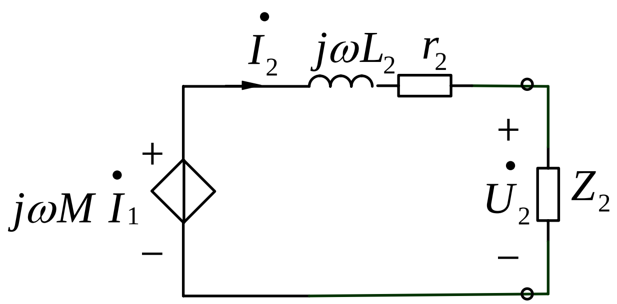
反映阻抗

$$Z_{22} = r_2 + j\omega L_2 + Z_2$$

- 计算反应阻抗
- 根据原边等效电路计算电流电压
- 根据副边等效电路计算

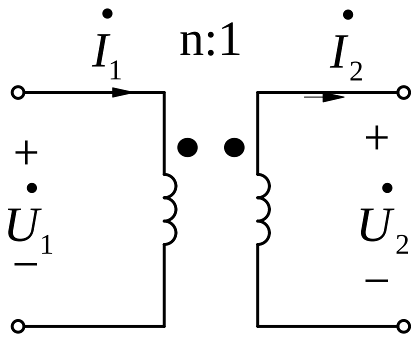



原边等效电路




副边等效电路

# 七、理想变压器

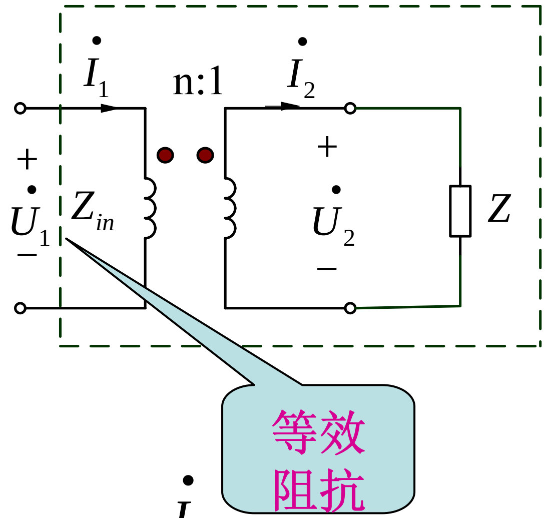



$$\begin{cases} \dot{U}_1 = n\dot{U}_2 \\ \dot{I}_2 = n\dot{I}_1 \end{cases}$$

伏安关系

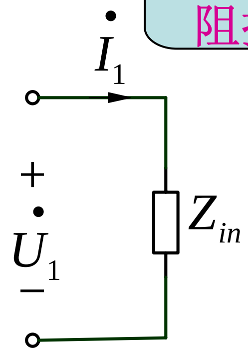
注意伏安关系的正负

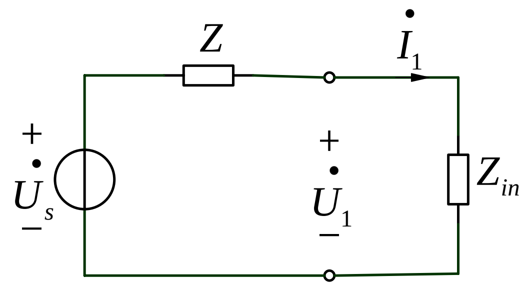
电流为零或电压为零！！！




计算等效阻抗

$$Z_{in} = n^2 Z$$



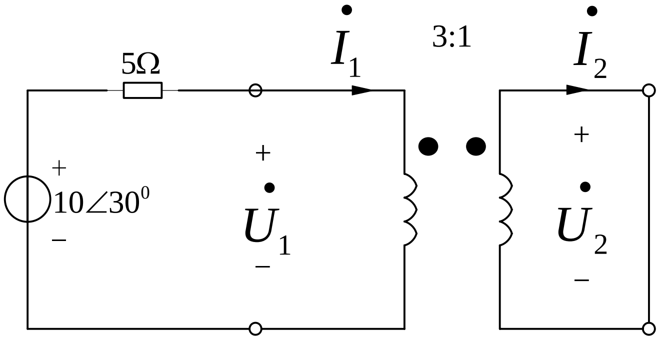



原边等效电路

根据原边等效电路计算电流，根据
理想变压器伏安关系计算副边电压电流。

# 七、理想变压器

## 副边短路

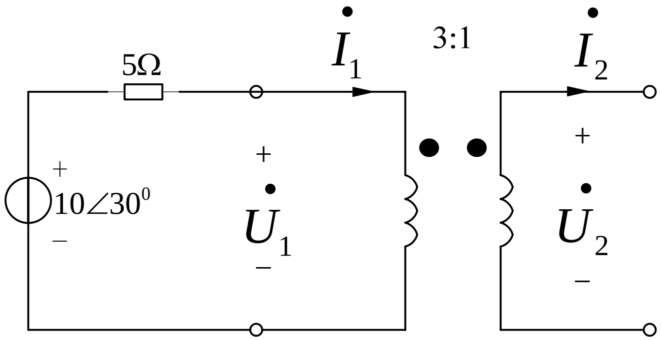



$$\dot{U}_2 = 0 \qquad \dot{U}_1 = 3\dot{U}_2 = 0$$

$$\dot{I}_1 = \frac{10\angle 30^0}{5} = 2\angle 30^0$$

$$\dot{I}_2 = 3\dot{I}_1 = 6\angle -30^0$$

## 副边开路




$$\dot{I}_2 = 0 \qquad \dot{I}_1 = 0$$

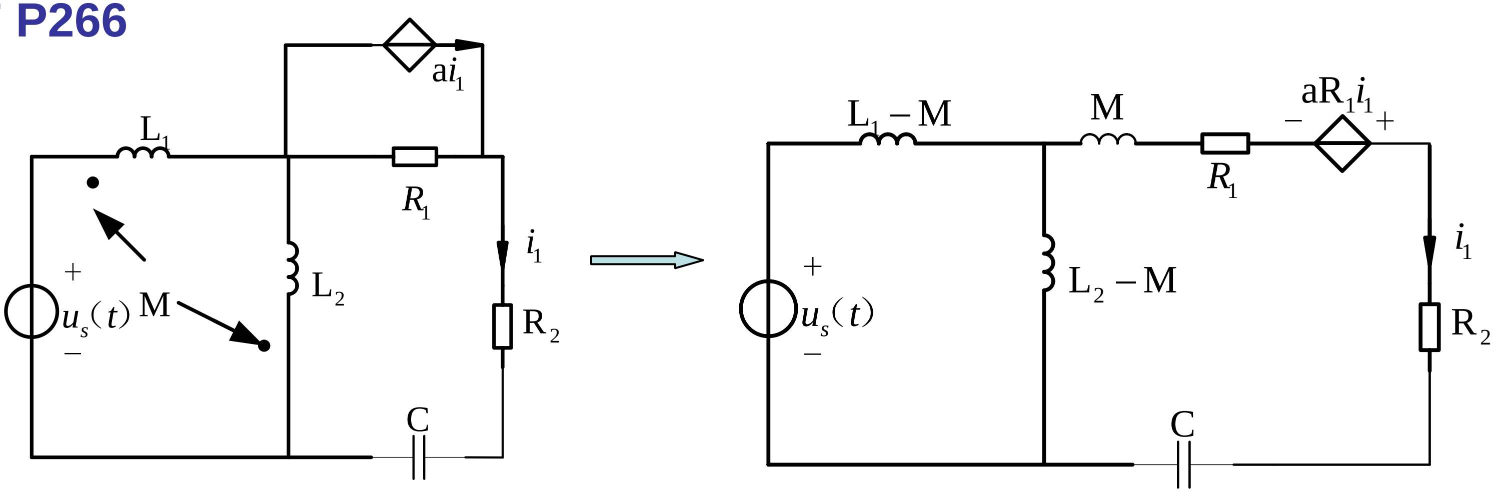
$$\dot{U}_1 = 10\angle 30^0$$

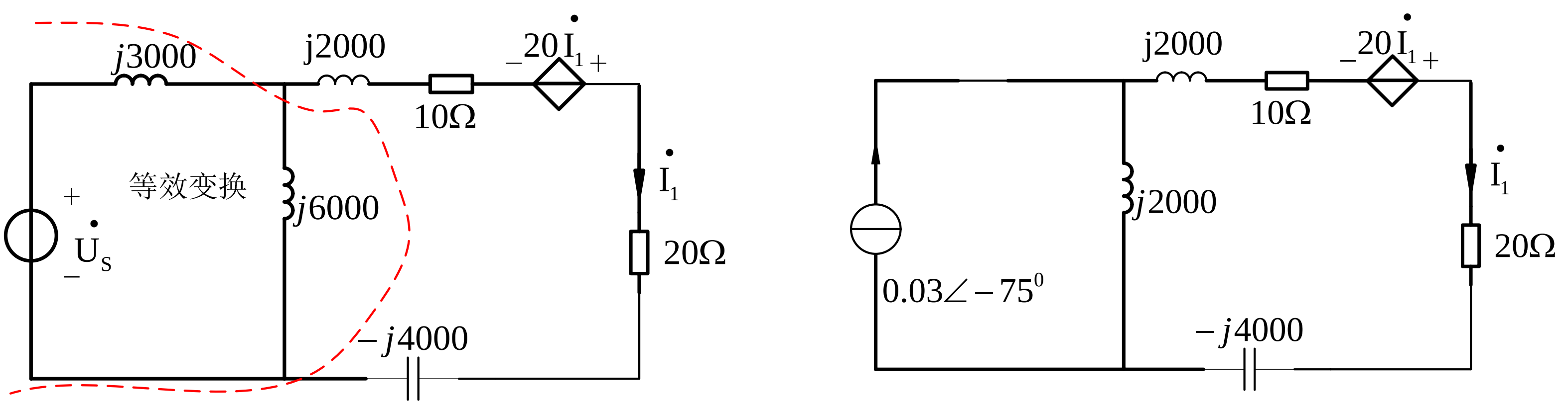
$$\dot{U}_1 = 3\dot{U}_2 \Rightarrow \dot{U}_2 = \frac{1}{3}10\angle 30^0$$

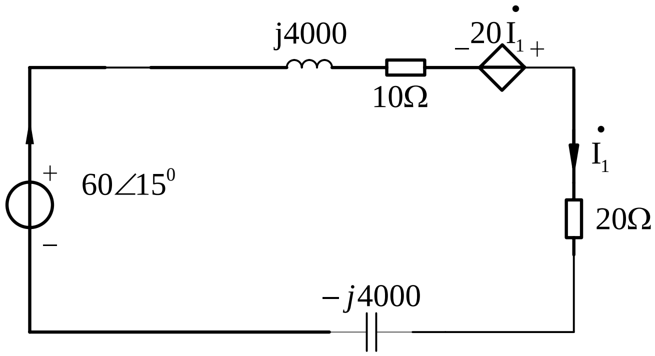
# 八、耦合电感问题分析

## 一般耦合电感电路，进行去耦等效后进行计算

**9-17 P266**





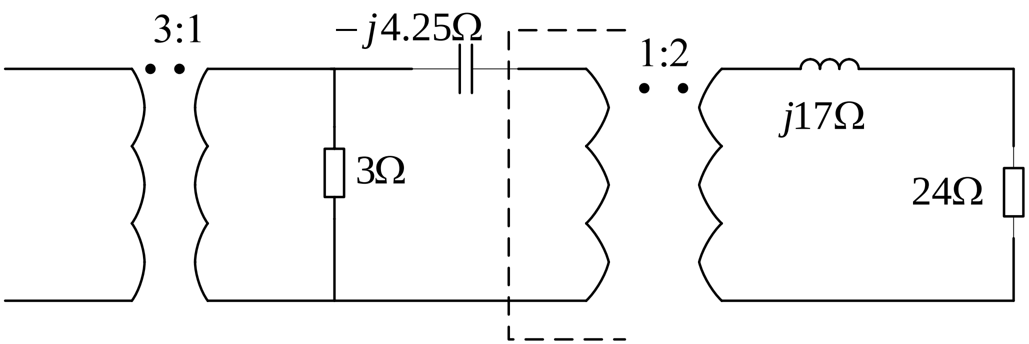

$$\dot{I}_1 = \frac{60\angle 15^0}{30-20} = 6\angle 15^0 A$$

$$P_{20i} = 6\times 20\times 6 = 720W$$

$$P_R = 36\times 30 = 1080W$$

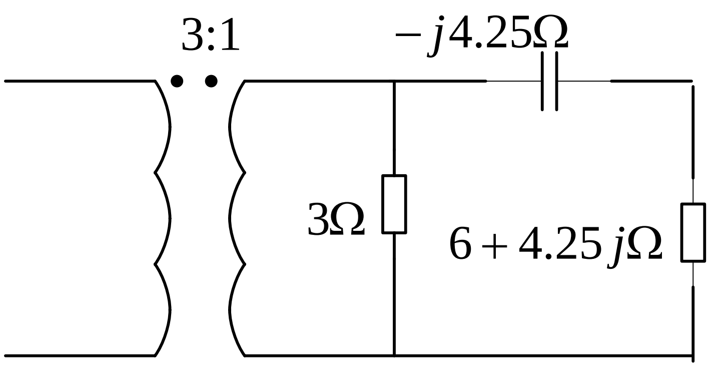
$$P_{us} = 1080 - 720 = 360W$$

# 求端口的输入阻抗




解

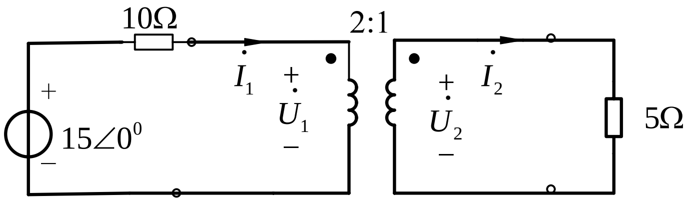
$$Z_{eq1}=\left(\frac{1}{2}\right)^2\times(24+17\mathrm{j})=6+4.25j$$




$$Z_{eq2}=\frac{(6+4.25j-4.25j)\times 3}{(6+4.25j-4.25j)+3}=2\Omega$$

$$Z_{eq}=3^2\times Z_{eq2}=9\times 2=18\Omega$$

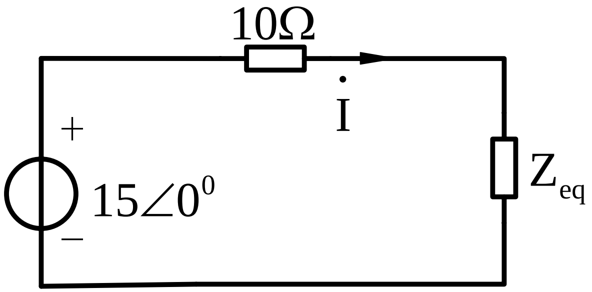
# 求电阻功率。




理想变压器副边开路与短路？？？

1）计算等效阻抗 $Z_{eq}=2^2\times5=20\Omega$

2）变压器原边等效电路




$$I=\frac{15}{20+10}=0.5\text{A}$$

$$P=I^2\times R=0.5^2\times20=5\text{W}$$

功率守恒

或 $P=I_2{}^2\times R=(0.5\times2)^2\times5=5\text{W}$

---

$$\begin{cases}\dot{U}_1=2\dot{U}_2\\ \dot{I}_2=2\dot{I}_1\end{cases}$$

双口网络参数方程

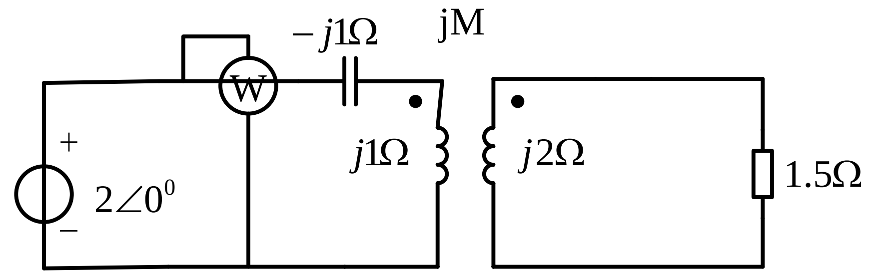
$$\begin{cases}10\dot{I}_1+\dot{U}_1=15\angle0^0\\ \dot{U}_2=5\dot{I}_2\end{cases}$$

端口伏安关系

$$\dot{I}_2=1\angle0^0\,A$$

$$P=I^2\times R=1^2\times5=5\text{W}$$

## 9-21 求互感系数M



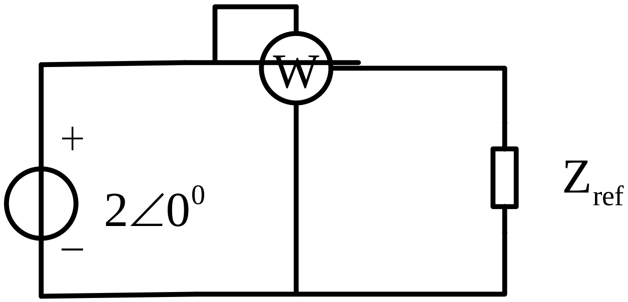

变压器副边开路与短路？？？

1）计算原边和副边自阻抗 $Z_{22}=1.5+2j$ $\quad Z_{11}=1j-1j=0$

2）计算反映阻抗 $Z_{ref}=\dfrac{(\omega M)^2}{Z_{22}}=(\omega M)^2\times\left(\dfrac{3-4j}{5}\right)$

$$tg\theta=-\frac{4}{3} \qquad \theta=-53^0$$

3）变压器等效电路




$$P=2\times I\times\cos(-53)=24W$$

$$I=\frac{24}{2\times0.6}=20A$$

$$P=I^2\times R=I^2\times(\omega M)^2\times\frac{3}{5}=24W$$

$$M=50mH$$

# 第八章 正弦稳态电路的相量分析

## 一、复数

$$A = a + bj = |A|\angle\theta = |A|(\cos\theta + j\cos\theta)$$

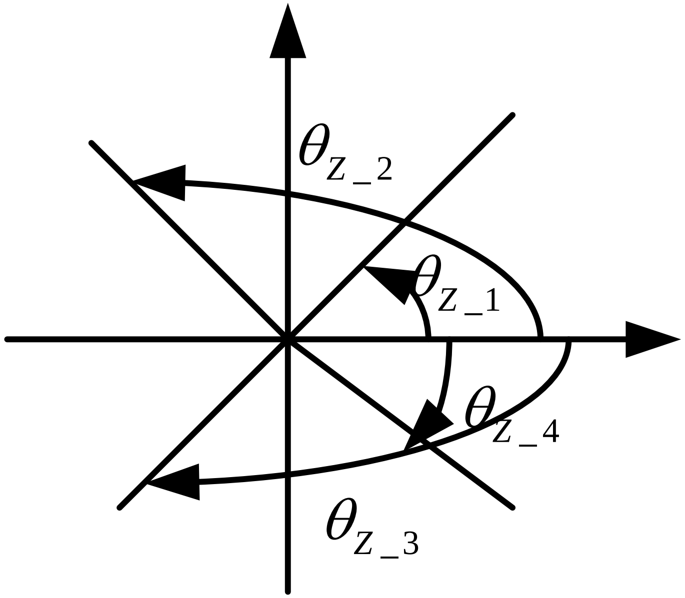
$$|A| = \sqrt{a^2 + b^2}$$

$$\theta = \mathrm{arc}tg\,\frac{b}{a}$$

1和4相限

$$\theta = \pm 180^0 + \mathrm{arc}tg\,\frac{b}{a}$$

2和3相限




$$a = |A|\cos\theta \qquad b = |A|\sin\theta$$

顺时针转动角度为负，逆时针转动角度为正

# 第八章 正弦稳态电路的相量分析

## 二、元件电压电流关系相量形式

$$\dot{U}_R = R \times \dot{I}_R \qquad \dot{U}_L = j\omega l \times \dot{I}_l$$

$$\dot{U}_C = -j\frac{1}{\omega C} \times \dot{I}_C \qquad \dot{U}_Z = (R + jX)\dot{I}_Z$$

$$Z = R + jX = |Z|\angle\theta_Z \qquad \theta_Z = arctg\left(X/R\right)$$

$$X = |Z|\sin\theta_Z \qquad R = |Z|\cos\theta_Z$$

## 三、 基尔霍夫相量形式

$$\sum \dot{U}_k = 0 \qquad \sum \dot{I}_k = 0$$

## 四、两类约束的相量图表示

**相量变换**

复数的模为正弦电压或电流的有效值或幅值

复数的辐角为正弦量的初相位

阻抗三角形
电压三角形
电流三角形
功率三角形

# 五、阻抗串并联

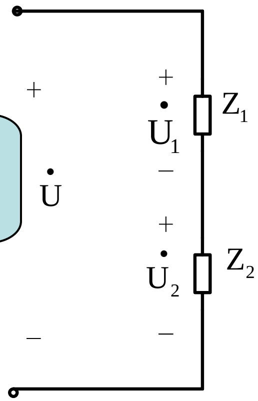
最简单阻抗----
单一原件

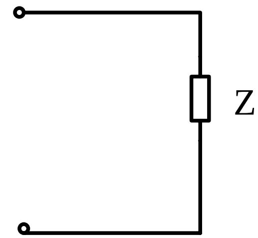
$$Z = Z_1 + Z_2$$

$$\dot{U}_2 = \frac{Z_2}{Z_1 + Z_2}\dot{U}$$

分压公式

$$\dot{U}_1 = \frac{Z_1}{Z_1 + Z_2}\dot{U}$$




不同性质的阻抗串联，阻抗模变小。

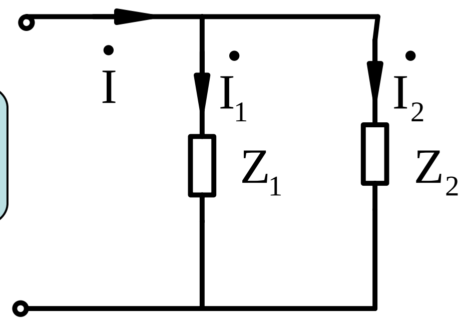
同性质阻抗串联阻抗模可以变大，也可以变小。

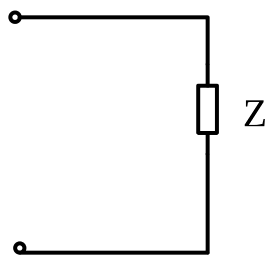
$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$\dot{I}_1 = \frac{Z_2}{Z_1 + Z_2}\dot{I}$$

分流公式

$$\dot{I}_2 = \frac{Z_1}{Z_1 + Z_2}\dot{I}$$



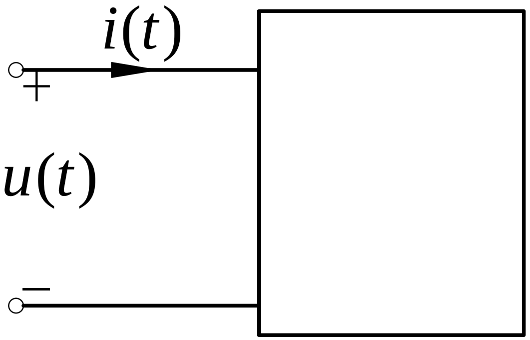



# 六、功率

$$P = UI\cos(\theta_u - \theta_i) = UI\lambda$$

功率因数

$$Q = UI\sin(\theta_u - \theta_i)$$




$$\begin{cases} S = UI \\ P = S\cos\theta \\ Q = S\sin\theta \end{cases}$$

功率因数角

$$\tilde{S} = \dot{U}\,\dot{I}^{*} = UI\cos\theta + jUI\sin\theta = P + Qj$$

电流的共轭复数

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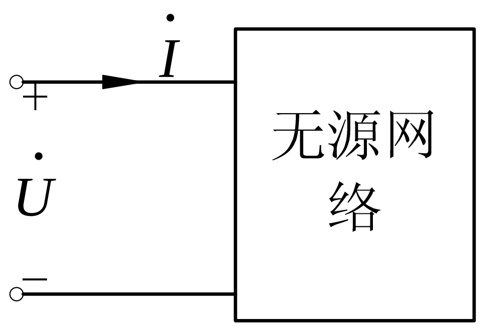
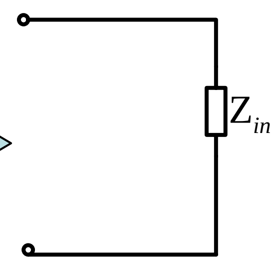
无源网络等效阻抗



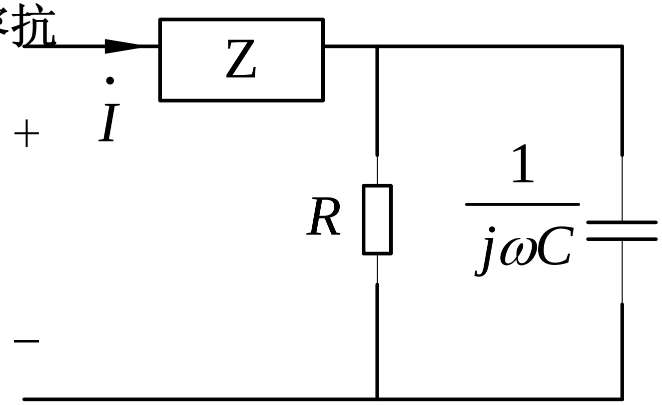

列写端口电压电流相量方程

$$Z_{in} = \frac{\dot{U}}{\dot{I}}$$



已知电源电压 $\dot{U}=165\angle 0^0$ 电源提供的复功率为 $\tilde{S}=1650-j825$

负载**Z**吸收的复功率为 $\tilde{S}_1=1250-j625$ 求电阻和容抗




解：**1**） $\tilde{S}=\dot{U}\times\dot{I}^* \Rightarrow 1650-j825=165\angle 0^0\,\dot{I}^*$

$$\dot{I}^*=\frac{1650-j825}{165\angle 0^0}=10-j5A$$

**2**）

$$1250-j625=\dot{U}_Z\dot{I}^* \qquad \dot{U}_Z=\frac{1250-j625}{10-j5}=125\angle 0^0$$

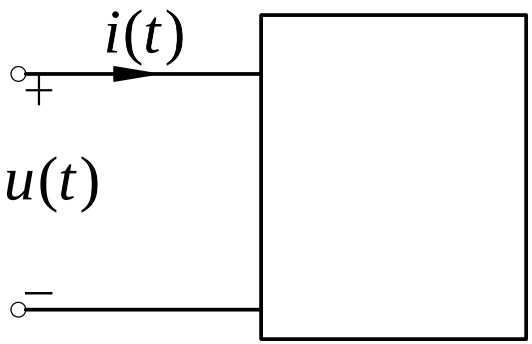
$$\dot{U}_R=\dot{U}-\dot{U}_Z=165\angle 0^0-125\angle 0^0=40\angle 0^0$$

$$1650-j825=1250-j625+\tilde{S}_2$$

$$\tilde{S}_2=400-j200 \qquad P_R=400 \quad Q_C=200$$

$$P_R=\frac{U_R^2}{R}\Rightarrow R=\frac{400}{1600}=0.25\Omega \quad Q_C=U_R^2\omega C\Rightarrow \omega C=\frac{200}{1600}=0.125$$

# 8-1 求串联最简等效电路及参数




$$u = 10\sqrt{2}\sin(t + 10^0)$$

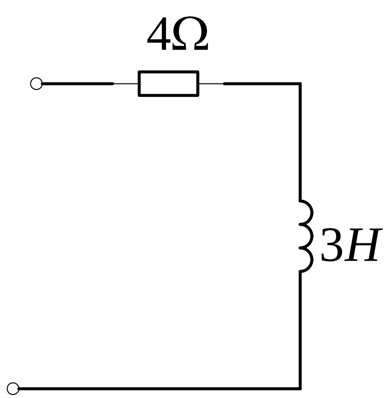
$$i = 2\sqrt{2}\cos(t - 117^0)$$

$$\dot{U} = 10\angle 10^0 V$$

$$\dot{I} = 2\angle -27^0 A$$

$$Z = \frac{\dot{U}}{\dot{I}} = \frac{10\angle 10^0}{2\angle -27^0} = 4 + j3\Omega$$

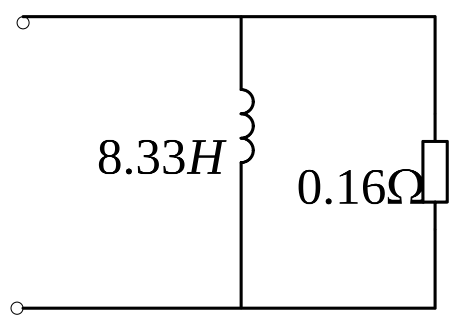
$R = 4\Omega$ $\quad\omega L = 3\Omega$ $\quad L = 3H$



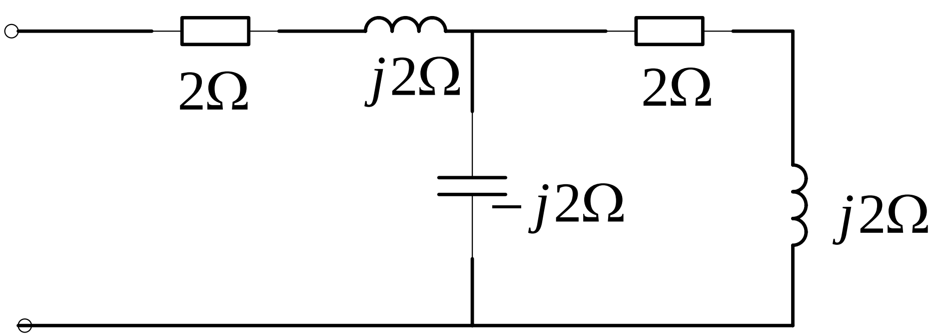

$$Y = \frac{\dot{I}}{\dot{U}} = \frac{2\angle -27^0}{10\angle 10^0} = 0.16 - 0.12jS$$

$$G = 0.16S$$

$$\frac{1}{\omega L} = 0.12 \qquad L = 8.33H$$



## 8-1 求等效阻抗



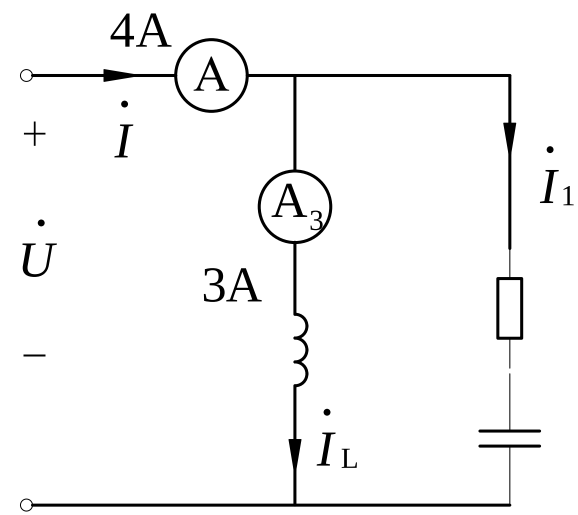

$$Z_3 = 2 + j2\Omega \qquad Z_2 = -j2\Omega$$

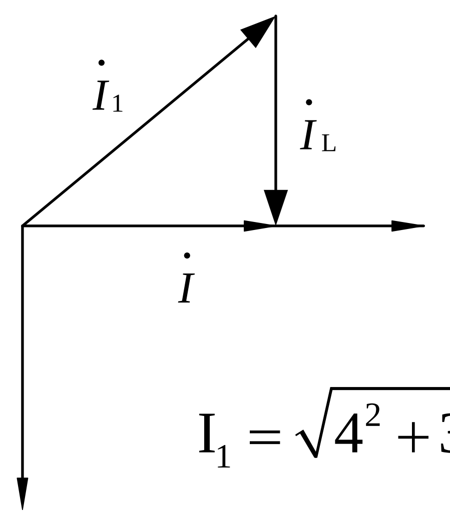
$$Z_1 = 2 + j2\Omega$$

$$Z_3 // Z_2 = \frac{Z_2 \times Z_3}{Z_3 + Z_2} = \frac{-j2 \times (2 + j2)}{-j2 + 2 + j2} = 2 - j2$$

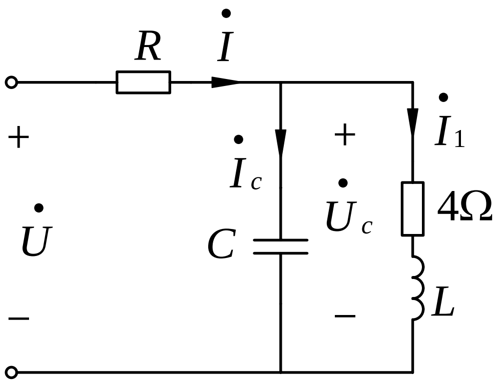
$$Z = Z_3 // Z_2 + Z_1 = 2 - j2 + 2 + j2 = 4\Omega$$

## 已知电压电流同相位，求电流I1




$$I_1 = \sqrt{4^2 + 3^2} = 5A$$



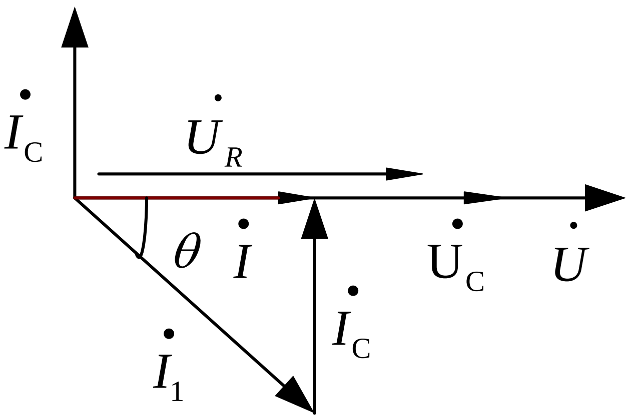

$I_1 = 5A \quad I_c = 3A \quad U = 65V$ 端口电压电流同相

**求R,L,C ,w=3000rad。**

解 令电容电压为参考，则 $\dot{U}_c = U_c\angle 0^0 V$

$$I = \sqrt{I_1^2 - I_C^2} = \sqrt{25-9} = 4A \qquad \theta = arctg\frac{3}{4} = 37^0$$

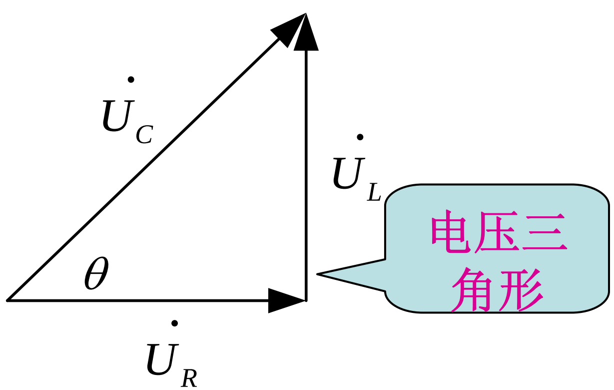
$$\frac{\omega L}{4} = tg37^0 \qquad \omega L = tg37^0 \times 4 = 3 \qquad \omega = 1mH$$



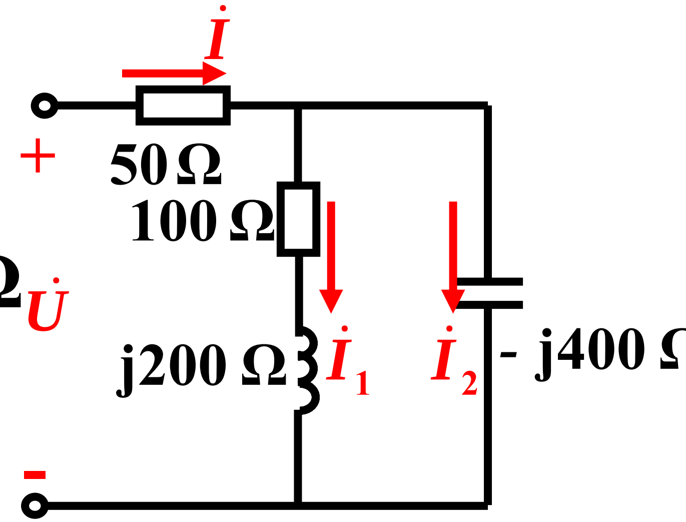

$$U_{4\Omega} = 4\times 5 = 20V \qquad U_C = \frac{20}{\cos 37^0} = 25V$$

$$U_R = 65 - 25 = 40V \qquad R = \frac{40}{4} = 10\Omega$$

$$\frac{1}{\omega C} = \frac{40}{3} \quad C = \frac{3}{3000\times 25} = 40uF$$



# 计算图中所示的电流




$$\dot{U} = 220\angle 0^\circ \text{ V}$$

$$Z_1 = R_1 + \text{j}X_L = (100 + \text{j}1200)\ \Omega$$

$$Z_2 = -\text{j}X_C = -\text{j}\,140\ \Omega$$

$$Z = [50 + \frac{(100+\text{j}200)(-\text{j}400)}{100+\text{j}200-\text{j}400}] = (50 + 320 + \text{j}240)$$

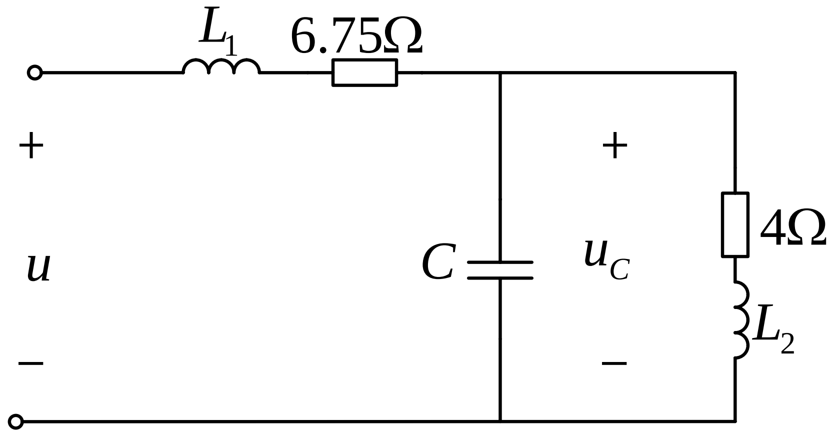
$$= 440\angle 33^\circ \Omega$$

$$\dot{I} = \frac{\dot{U}}{Z} = \frac{220\angle 0^\circ}{440\angle 33^\circ} \text{A} = 0.5\angle -33^\circ \text{A}$$

$$\dot{I}_1 = \frac{Z_2}{Z_1 + Z_2}\dot{I} = \frac{-\text{j}400}{100+\text{j}200-\text{j}400} \times 0.5\angle -33^\circ \text{A}$$

$$= 0.89\angle -59.6^\circ \text{A}$$

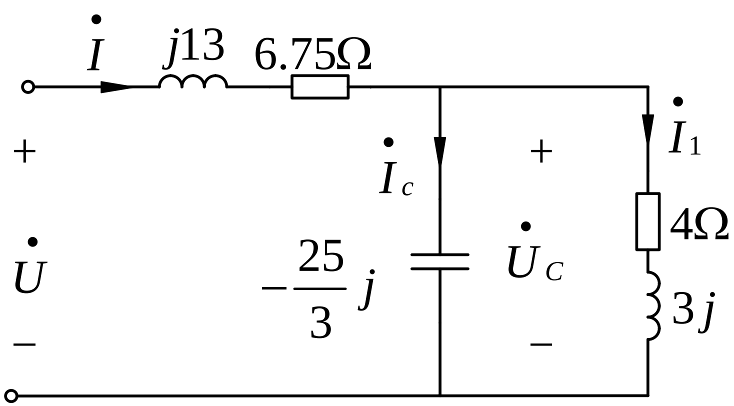
已知**Uc=50V,L1=6.5H,L2=1.5H,**电源角频率为**2rad/s, C=0.06F**，求端口电压和端口电流的有效值并计算电路吸收的有功功率，无功功率，复功率。




（1）画相量模型

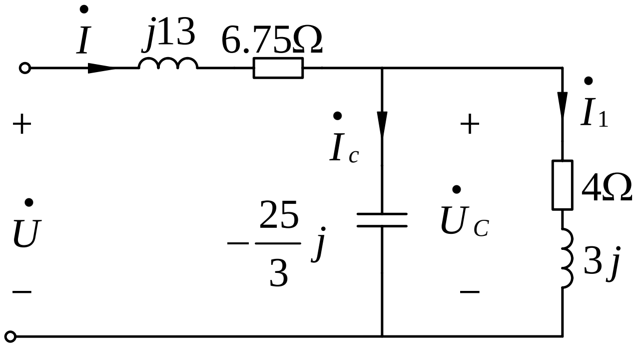
$$\omega L_1 = 2\times 6.5 = 13\Omega \quad \omega L_2 = 2\times 1.5 = 3\Omega$$

$$\frac{1}{\omega C} = \frac{1}{2\times 0.06} = \frac{25}{3}\Omega$$




（2）令电容电压为参考相量

$$\dot{U}_C = 50\angle 0^0 V$$




$$\dot{U} = \dot{I}(6.75+13j)+\dot{U}_C$$
$$=54+104j+50$$
$$=104+104j=104\sqrt{2}\angle 45^0$$

端口电压电流相位差

$$\dot{I}_C=\frac{\dot{U}_C}{-25/3j}=\frac{50\angle 0^0}{25/3\angle -90^0}=6\angle 90^0$$

$$\dot{I}_1=\frac{\dot{U}_C}{4+3j}=\frac{50\angle 0^0}{5\angle 37^0}=10\angle -37^0$$

$$\dot{I}=\dot{I}_1+\dot{I}_C=10\angle -37^0+6\angle 90^0$$
$$=10\left(\cos-37+j\sin-37\right)+6j$$
$$=10\times 0.8-10\times 0.6j+6j=8$$

（**3**）

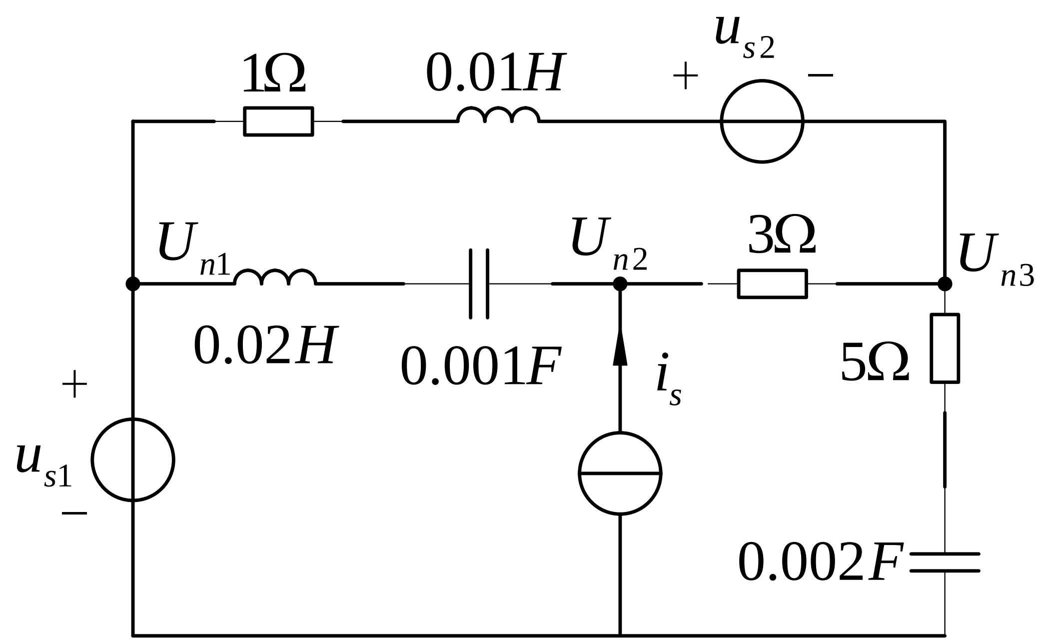
$$\theta=45-0=45^0$$
$$\lambda=\cos 45^0=0.707$$

$$P=UI\cos\theta=104\times 8\times\sqrt{2}/2=416\sqrt{2}W$$
$$Q=UI\sin\theta=104\times 8\times\sqrt{2}/2=416\sqrt{2}Va$$
$$S=UI=832VA$$
$$\tilde{S}=\dot{U}\overset{*}{\dot{I}}=104\sqrt{2}\angle 45^0\times 8$$
$$=416\sqrt{2}+416\sqrt{2}j$$

# 列写图所示电路节点电压方程

已知 $u_{s1}=5\sqrt{2}\sin(100t+30^0)$ $u_{s2}=10\sqrt{2}\sin(100t+60^0)$

$i_s=3\sqrt{2}\cos(100t-60^0)$




解：**1**）求电路参数

$$\omega L_1=100\times0.01=1\Omega$$

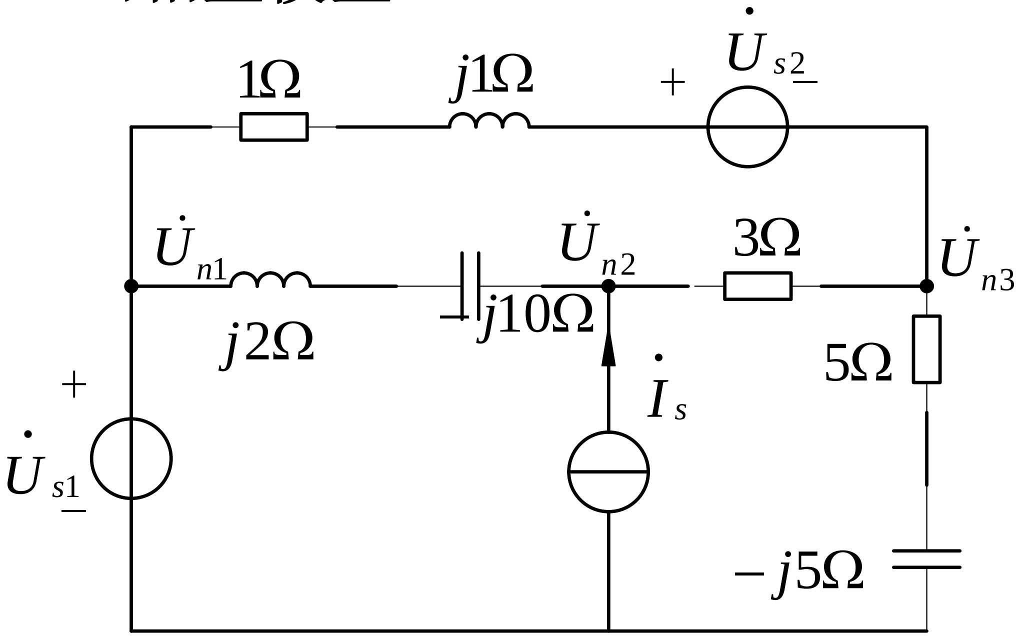
$$\omega L_2=100\times0.02=2\Omega$$

$$\frac{1}{\omega C_1}=\frac{1}{100\times0.001}=10\Omega$$

$$\frac{1}{\omega C_2}=\frac{1}{100\times0.002}=5\Omega$$

$$\dot{U}_{s1}=5\angle30^0\qquad\dot{U}_{s2}=10\angle60^0\qquad\dot{I}_s=3\angle30^0$$

2）画相量模型



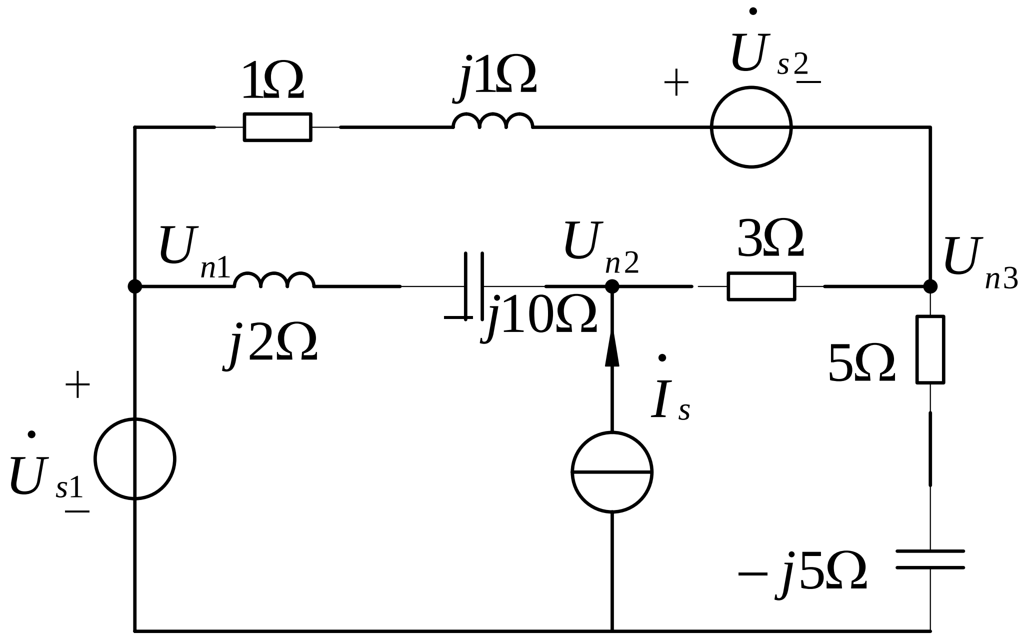

3）列写节点1方程

$$\dot{U}_{n1} = \dot{U}_{s1} = 5\angle 30^0$$

4）列写节点2方程

$$\left(\frac{1}{j2 - j10} + \frac{1}{3}\right)\dot{U}_{n2} - \frac{1}{3}\dot{U}_{n3} - \left(\frac{1}{j2 - j10}\right)\dot{U}_{n1} = 3\angle 30^0$$

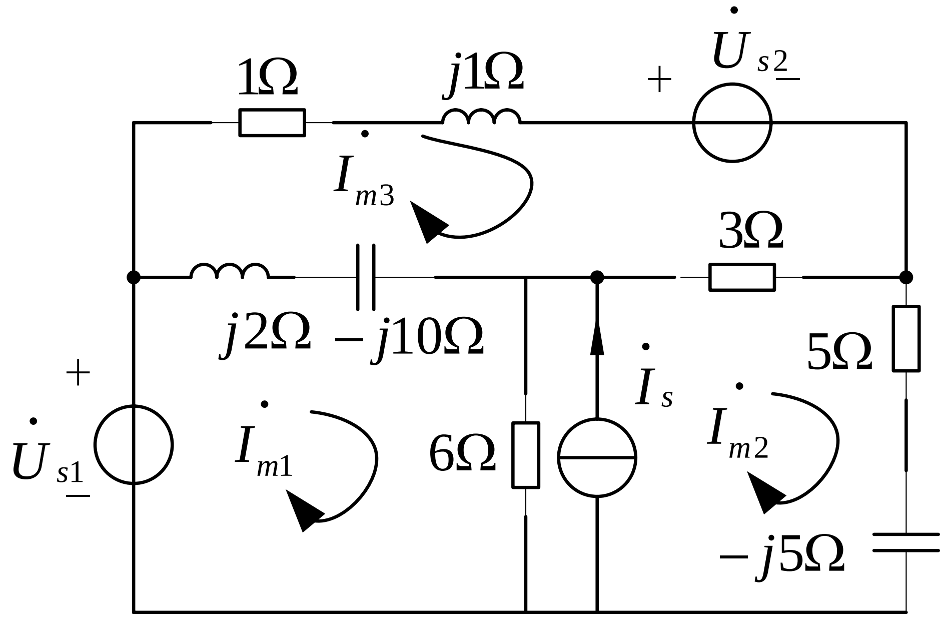
$$\left(\frac{1}{-j8} + \frac{1}{3}\right)\dot{U}_{n2} - \frac{1}{3}\dot{U}_{n3} + \frac{1}{j8}\dot{U}_{n1} = 3\angle 30^0$$




# 5）列写节点3方程

$$(\frac{1}{5-j5}+\frac{1}{3}+\frac{1}{1+j})\dot{U}_{n3}-\frac{1}{3}\dot{U}_{n2}-(\frac{1}{1+j})\dot{U}_{n1}=-\frac{10\angle 60^0}{1+j}=-\frac{10\angle 60^0}{\sqrt{2}\angle 45^0}=-5\sqrt{2}\angle 15^0$$

# 列写图所示电路网孔电流方程

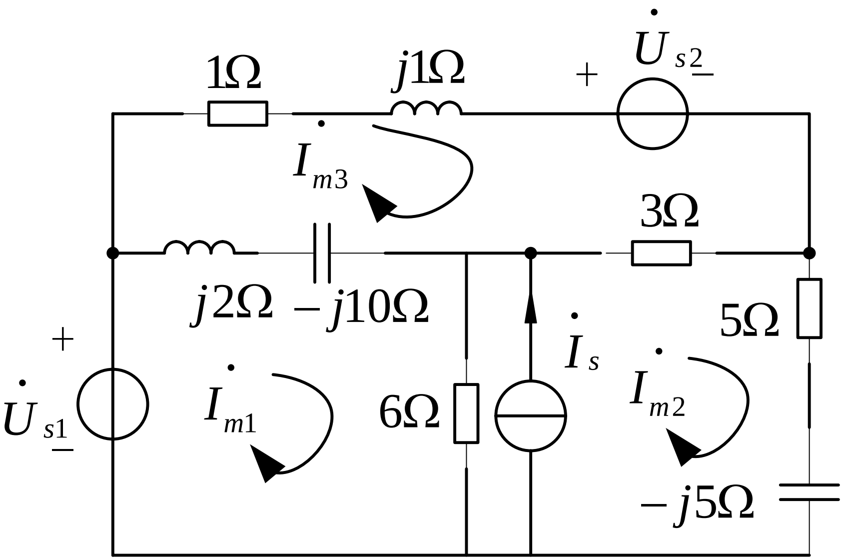



**1**）列写网孔**1**方程

$$(j2-j10+6)\dot{I}_{m1}-6\dot{I}_{m2}-(j2-j10)\dot{I}_{m3}=\dot{U}_{s1}-\dot{I}_3\times 6$$

$$(-j8+6)\dot{I}_{m1}-6\dot{I}_{m2}+(j8)\dot{I}_{m3}=5\angle 30^0-3\angle 30^0\times 6=-13\angle 30^0$$

# 列写图所示电路网孔电流方程




**2**）列写网孔**2**方程

$$(3-j5+5)\dot{I}_{m1}-6\dot{I}_{m1}-3\dot{I}_{m3}=\dot{I}_3\times 6$$

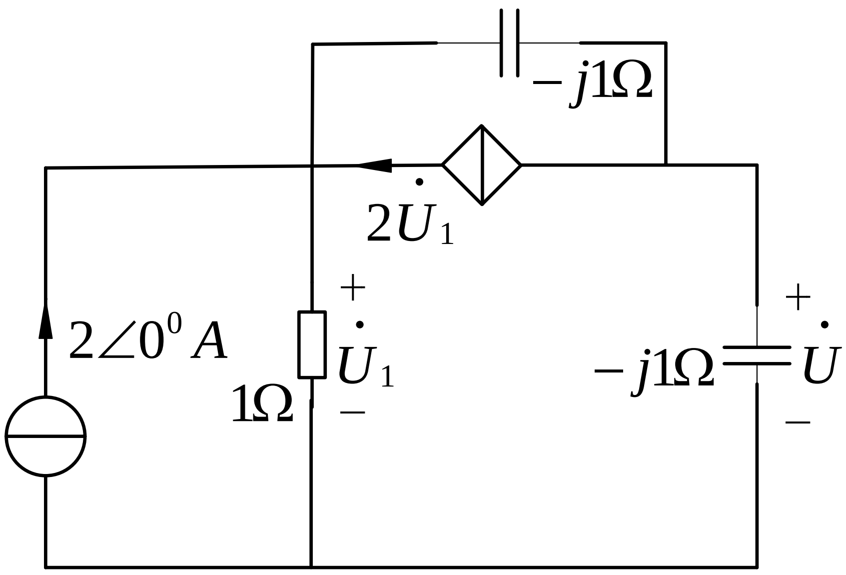
$$(8-j5)\dot{I}_{m1}-6\dot{I}_{m1}-3\dot{I}_{m3}=18\angle 30^0$$

**3**）列写网孔**3**方程

$$(3+j2-j10+1+j1)\dot{I}_{m3}-(j2-j10)\dot{I}_{m1}-3\dot{I}_{m2}=-\dot{U}_{s2}$$

$$(4-j7)\dot{I}_{m3}+j8\dot{I}_{m1}-3\dot{I}_{m2}=-10\angle 60^0$$

# 电路如图所示，利用戴维南定理求电压u

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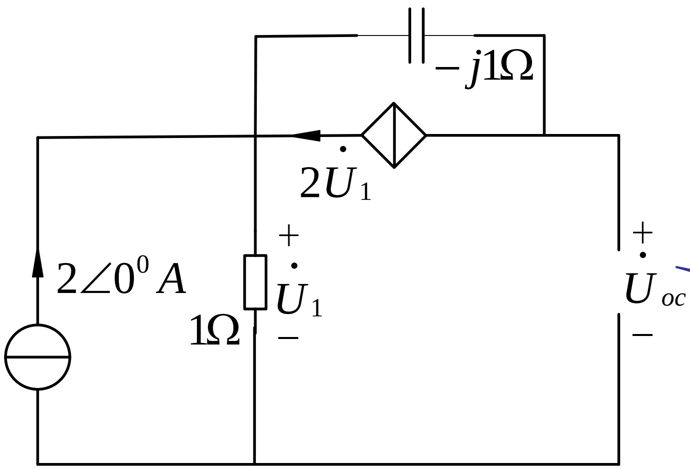
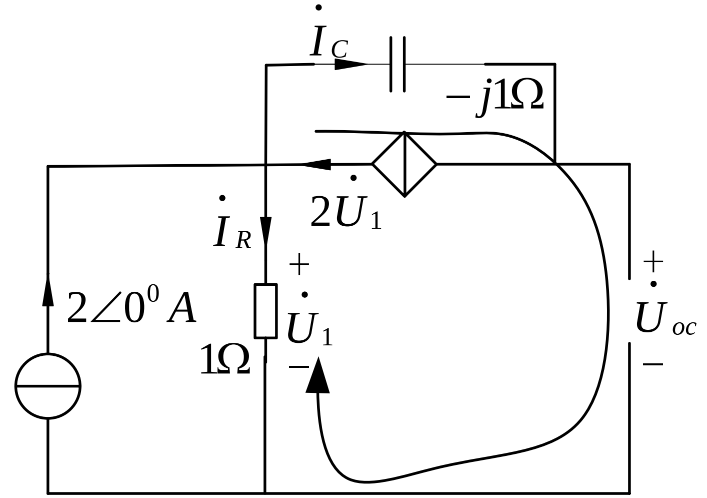
## 解：1）求开路电压



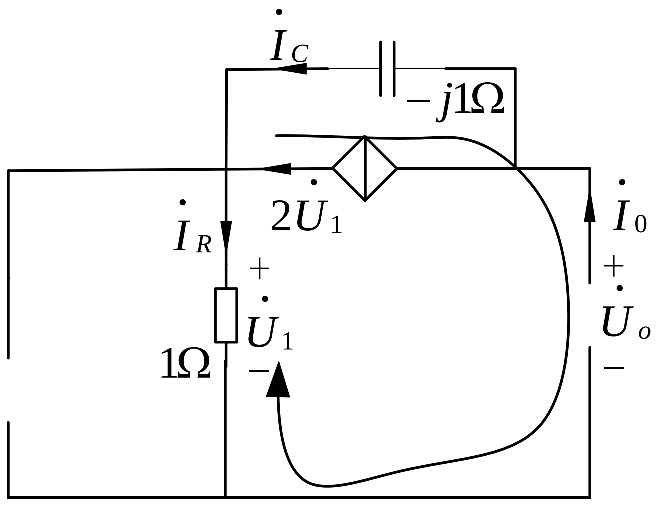

$$\dot{I}_c = 2\dot{U}_1 \qquad \dot{I}_R = 2\angle 0^0$$

$$\dot{U}_1 = 1\times \dot{I}_R = 2\angle 0^0$$

$$\dot{U}_{oc} = \dot{U}_1 - \dot{I}_c \times -j1 = 2 + j4$$

断开待求支路

## 2）等效阻抗



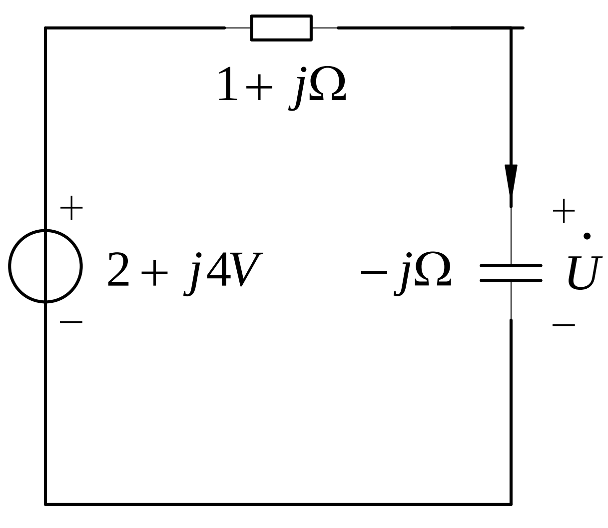

$$\dot{I}_0 = \dot{I}_c + 2\dot{U}_1 \qquad \dot{I}_R = \dot{I}_0 = \frac{\dot{U}_1}{1}$$

$$\dot{I}_c = -\dot{U}_1$$

$$\dot{U}_0 = \dot{I}_c \times -j1 + \dot{U}_1 = (1+j)\dot{U}_1$$

$$Z_{eq} = \frac{\dot{U}_0}{\dot{I}_0} = \frac{(1+j)\dot{U}_1}{\dot{U}_1} = 1+j\Omega$$

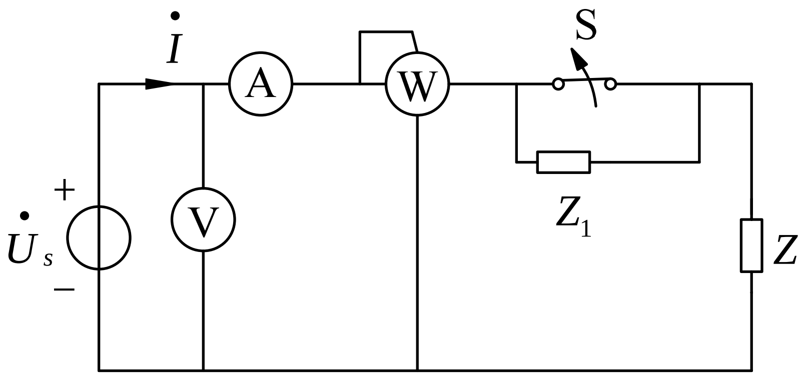
## 3）画等效电路并求解




$$\dot{U} = \frac{2+j4}{1+j-j} \times -j = 4-j2$$

$$= 2\sqrt{5}\angle\text{-}26.6^0\text{V}$$

**8-20**




已知开关闭合，**U=220V，P=1000W，I=10A,**开关打开， **U=220V，P=1600W，I=12A，**阻抗**Z1**为感性，求**Z1**和**Z**。

串联阻抗后电流变大，所以阻抗性质相反

**(1)** 令

$$Z = \mathrm{R+X}j \qquad Z_1 = \mathrm{R_1+X_1}j$$

**(2)** $P = I^2 R \qquad 1000 = 10^2 R \qquad R = 10\Omega$

$$Z = 10 - 19.6j$$

$$|Z| = \frac{U}{I} = \sqrt{R^2 + X^2} = 22 \qquad X = -19.6\Omega$$

**(3)** $1600 = 12^2 (R + R_1) \qquad R_1 = 1.11\Omega$

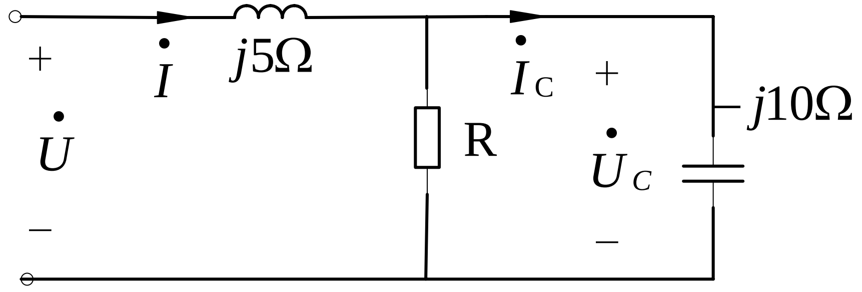
$$X_1 - 19.6 = \pm 14.58$$

$$|Z + Z_1| = \frac{U}{I} = \sqrt{(\mathrm{R+R_1})^2 + (X_1 + X)^2} = \frac{220}{12}$$

$$Z_1 = 1.11 + 5.02j$$

$$Z_1 = 1.11 + 34.18j$$

**8-2**




已知**C=0.02F，L=1H,**电路消耗的功率**P=10W**，求电路功率因数。

$$i_c(t)=\sqrt{2}\sin(5t+90^0)A$$

**(1)** $\omega L=5\times 1=5\Omega$ $\qquad \frac{1}{\omega C}=\frac{1}{5\times 0.02}=10\Omega$

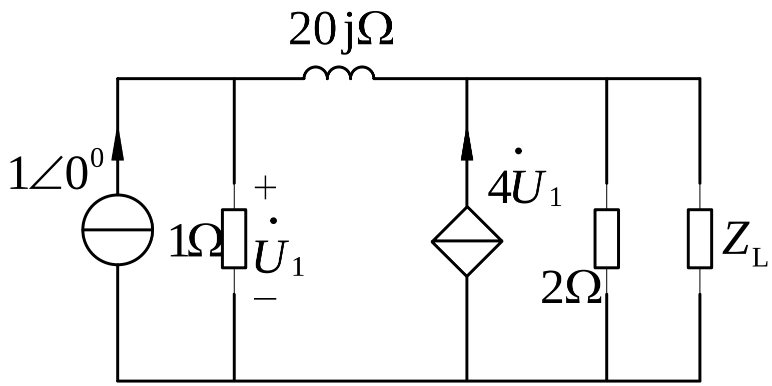
$$\dot{I}_c=1\angle 90^0\,A$$

**(2)** $\dot{U}_C=-j10\times 1\angle 90^0=10\angle 0^0\,V$ $\qquad P=\frac{U_c^2}{R}=10$ $\qquad R=\frac{100}{10}=10\Omega$

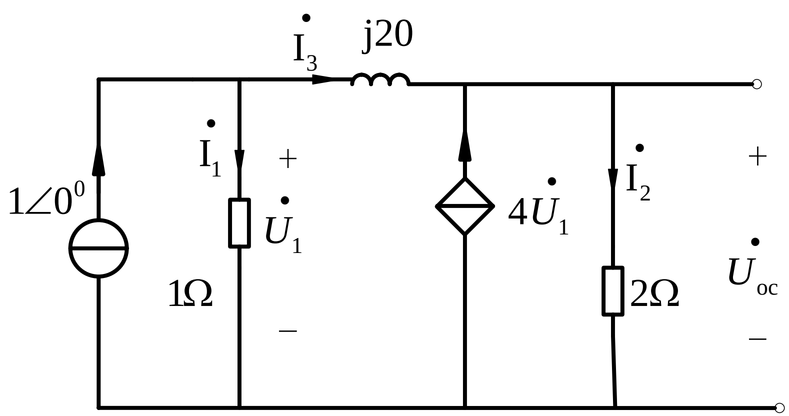
$$Z=j5+-j10//10=5j+5-5j\Omega=5\Omega$$

$$\lambda=\cos 0^0=1$$

**8-34** 正弦稳态电路中，负载**ZL**可以变化，问**ZL**为多少的时候其获得功率最大




解 **1**）求开路电压



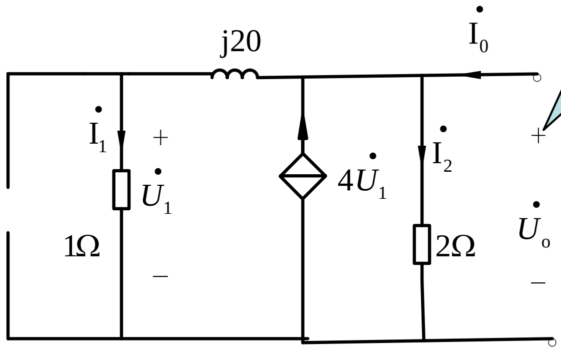

$$\dot{I}_3=\dot{I}_2-4\dot{U}_1 \quad \dot{U}_1+\dot{I}_3=1\angle 0^0$$

$$\dot{U}_1=(1-\dot{U}_1)\times 20j+2\times \dot{I}_2$$

$$\dot{U}_1=\frac{2+20j}{20j-5}=1\angle -20^0 V$$

$$\dot{U}_{oc}=2+6\angle -20^0=8\angle -15^0$$

解 2）求等效阻抗



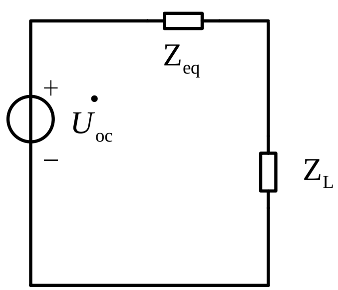

列写端口电压电流方程

$$\dot{U}_0 = \dot{U}_1 + \dot{U}_1 \times j20$$

$$\dot{I}_0 = \frac{\dot{U}_0}{2} - 4\dot{U}_1 + \dot{U}_1 = -2.5\dot{U}_1 + 10j \times \dot{U}_1$$

$$Z_{eq} = \frac{\dot{U}_0}{\dot{I}_0} = \frac{1 + j20}{-2.5 + 10j} = 1.858 - 0.565j$$

3）等效电路

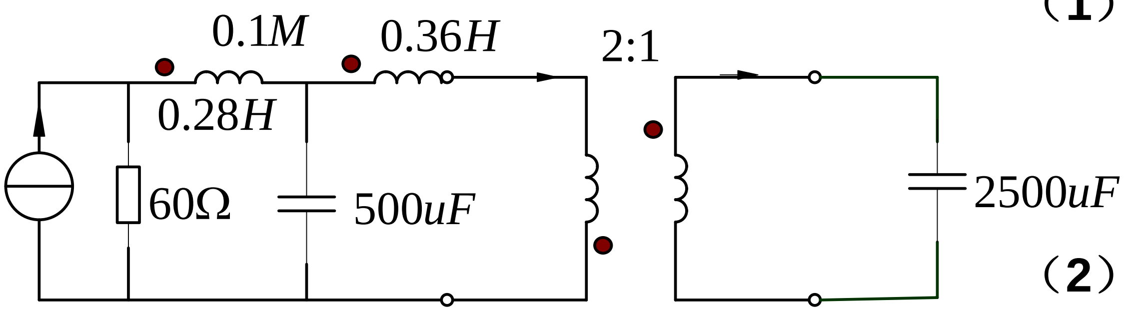



为等效阻抗的共轭复数

$$Z = Z_{eq}^* = 1.858 + 0.565j$$

$$P_{\max} = \frac{U_{oc}^2}{4 \times 1.858} = 8.11W$$

已知 $i_s = 4\sqrt{2}\sin(100t)A$ 求电容电压电流



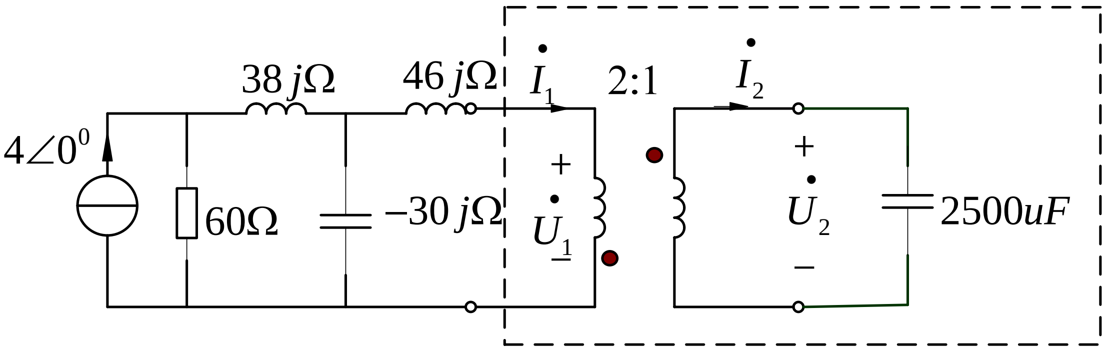

（1） $\omega(0.28+0.1) = 100\times0.38 = 38\Omega$

$\omega(0.36+0.1) = 100\times0.46 = 46\Omega$

$\omega M = 100\times0.1 = 10\Omega$

$\frac{1}{\omega C_1} = \frac{1}{100\times500\times10^6} = 20\Omega$

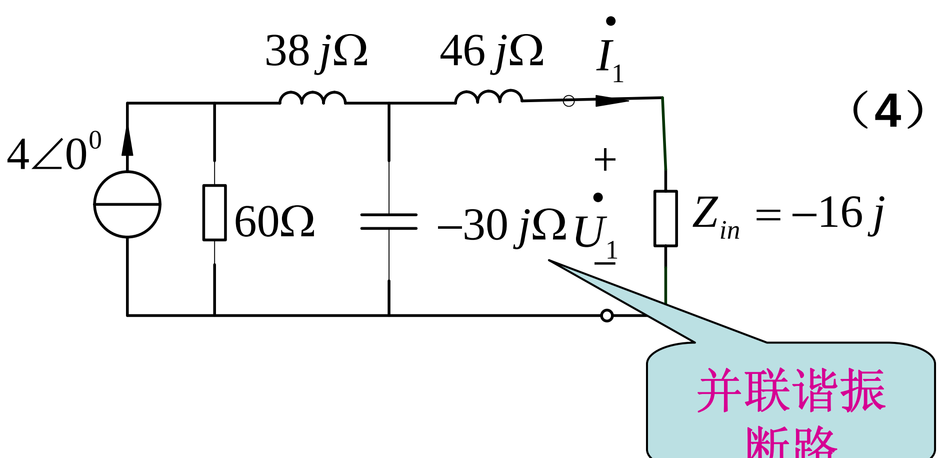
（2） $\frac{1}{\omega C_2} = \frac{1}{100\times2500\times10^6} = 4\Omega$




（3） $Z_{in} = 2^2\times(-4j) = -16j\Omega$

$46j - 16j = 30j\Omega$

$\dot{U}_1 = 4\angle0^0\times60 = 240\angle0^0$




（4） $\dot{I}_L = \frac{\dot{U}}{30j} = \frac{240\angle0^0}{30\angle90^0} = 8\angle-90^0$

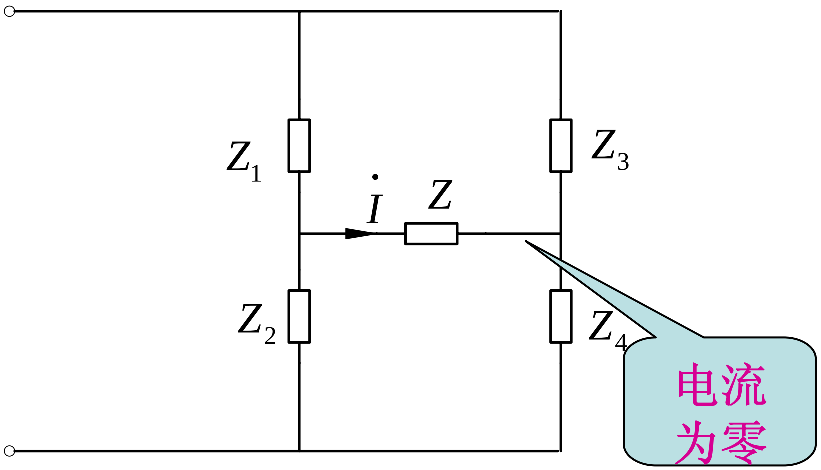
$\dot{U}_1 = Z_{in}\dot{I}_L = 8\angle-90^0\times16\angle-90^0 = 128\angle-180^0$

（5） $\dot{U}_c = -0.5\dot{U}_1 = 64\angle0^0$

$\dot{I}_C = -2\dot{I}_L = 16\angle90^0$

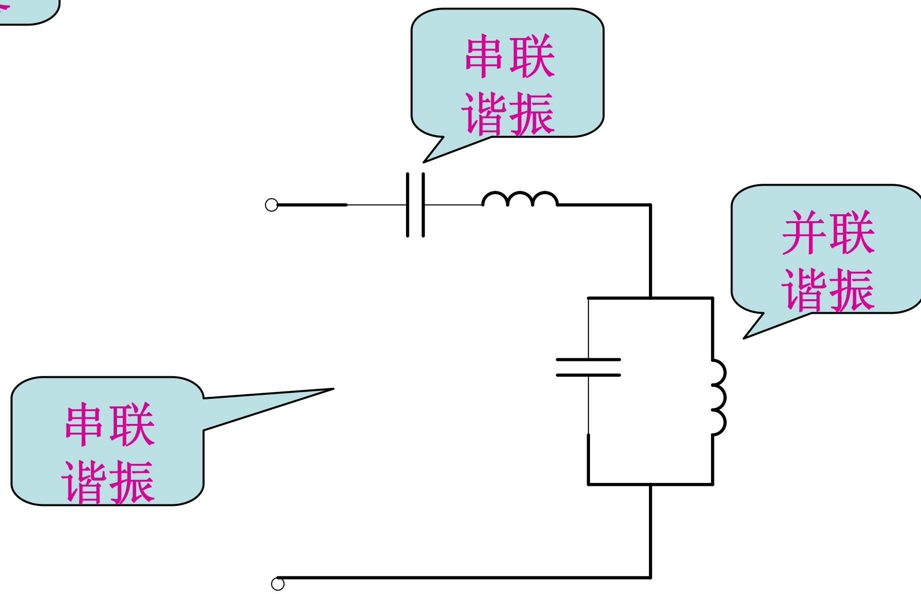
伏安关系

# 交流电桥平衡




$$Z_1 \times Z_4 = Z_2 \times Z_3$$

多个谐振频率

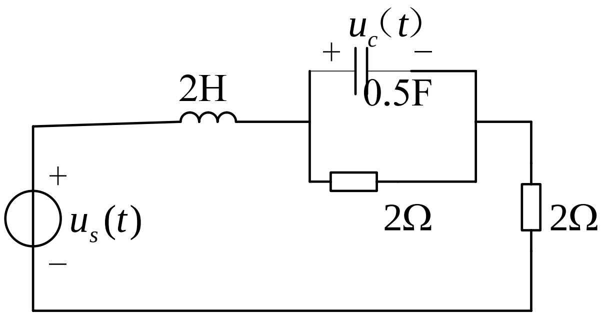


# 第六章 线性动态电路的时域分析

## 一、输入输出方程

根据两类约束 $i_c = C\dfrac{du_c}{dt}$ $u_l = \mathrm{L}\dfrac{di_l}{dt}$

**6-1**




$$i_1 = \frac{u_c(t)}{2} \qquad i_c = c\frac{du_c(t)}{dt}$$

$$i_l = i_1 + i_c = \frac{u_c(t)}{2} + c\frac{du_c(t)}{dt}$$

$$u_s = L\frac{di_l(t)}{dt} + u_c(t) + 2\times i_l$$

$$u_s = L\frac{d\left(\frac{u_c(t)}{2} + c\frac{du_c(t)}{dt}\right)}{dt} + u_c(t) + 2\times\frac{u_c(t)}{2} + 2\times c\frac{du_c(t)}{dt}$$

$$u_s = \frac{du_c(t)}{dt} + \frac{d^2u_c(t)}{dt} + 2u_c(t) + \frac{du_c(t)}{dt}$$

$$= 2\frac{du_c(t)}{dt} + \frac{d^2u_c(t)}{dt} + 2u_c(t)$$

# 二、三要素法

1）根据换路前稳态电路求 $u_c(0_-)$ $i_L(0_-)$ 根据换路定则确定

2）根据零正时刻电路求 $y(0_+)$

$$u_c(0_+) = u_c(0_-)$$

3）根据换路后稳态电路求 $y(\infty)$

$$i_L(0_+) = i_L(0_-)$$

4）将换路后的电路化为无源电路，求解储能元件两端的等效电阻 $R_{eq}$ 并求时间常数。

5）代入三要素公式

待求电压或电流的初始值

$$y(t) = y(\infty) + [y(0_+) - y(\infty)]e^{\frac{-t}{\tau}}$$

待求电压或电流

待求电压或电流的稳态值

电路时间常数

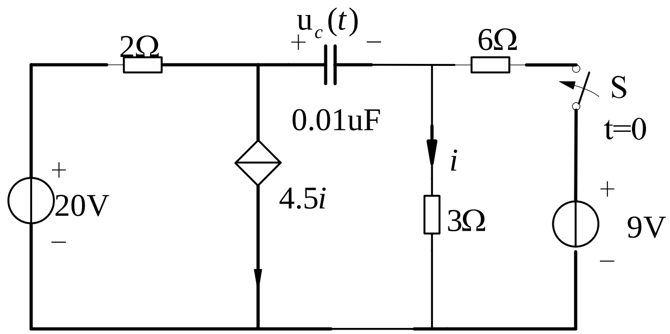
电阻电容电路

$$\tau = R_{eq} \times C$$

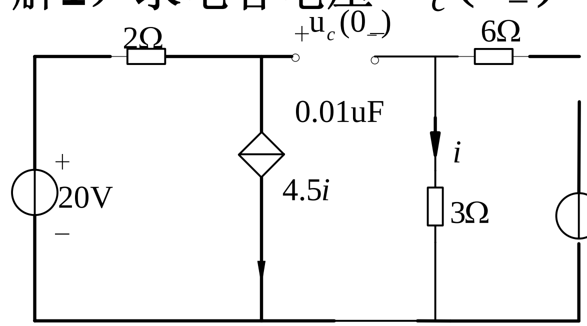
电阻电感电路

$$\tau = \frac{l}{R_{eq}}$$

例：求开关闭合后电容电压和电流



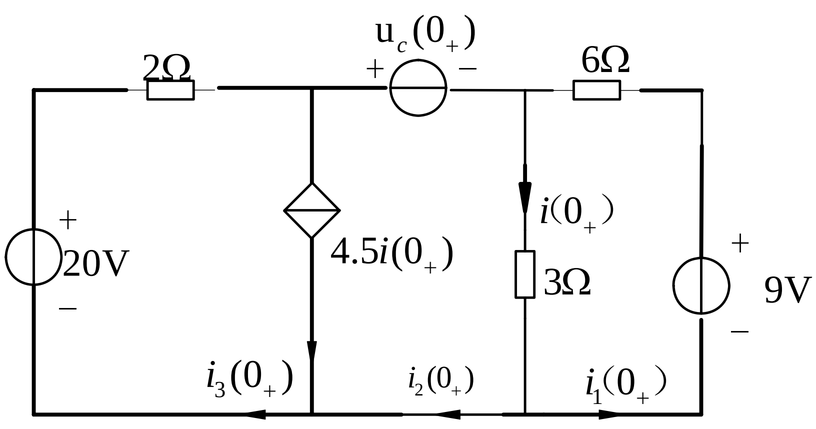

解1）求电容电压 $u_c(0_-)$




换路前直流稳态电路

$$u_c(0_-)=20V$$

2）求 $u_c(0_+)$ $i(0_+)$




$$u_c(0_+)=u_c(0_-)=20V$$

零正时刻电路

$$i_1(0_+)=\left(9-3i(0_+)\right)/6$$

$$i_2(0_+)=i(0_+)-i_1(0_+)=1.5i(0_+)-1.5$$

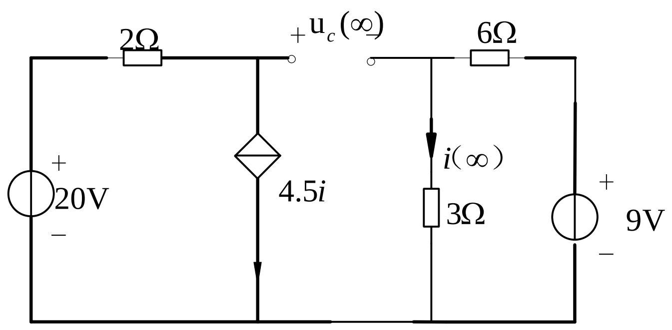
$$i_3(0_+)=4.5i(0_+)+i_2(0_+)=6i(0_+)-1.5$$

$$-20+2i_3(0_+)+20+3i(0_+)=0$$

$$12i(0_+)+3i(0_+)=3$$

$$i(0_+)=0.2A$$

3）求 $u_c(\infty)$ $i(\infty)$



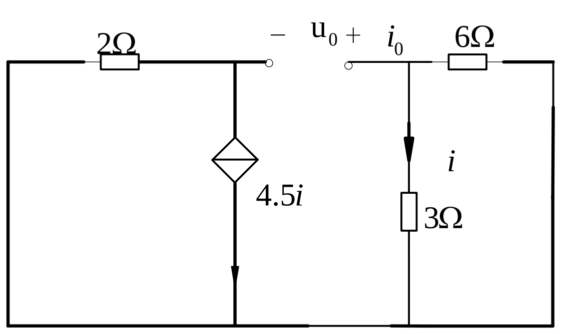

$$u_c(\infty)+i(\infty)\times 3-20+4.5i(\infty)\times 2=0$$

$$u_c(\infty)=8V$$

$$i(\infty)=\frac{9}{6+3}=1A$$

换路后直流稳态电路

4）求输入电阻




$$i_0=i+\frac{3i}{6}=1.5i \qquad u_0=3i+(i_0+4.5i)\times 2=15i$$

$$R=\frac{u_0}{i_0}=10\Omega \qquad \tau=R\times C=10\times 0.01=0.1S$$

换路后无源电路

5）代入三要素公式求响应

$$u_c(t)=u_c(\infty)+\left[u_c(0_+)-u_c(\infty)\right]e^{\frac{-t}{\tau}}=8+12e^{-10t}V$$

$$i(t)=i(\infty)+\left[i(0_+)-i(\infty)\right]e^{\frac{-t}{\tau}}=1-0.8e^{-10t}A$$

## 三、一阶动态电路的线性特性

全响应=零输入响应+零状态相应

零输入响应

当 $u_c(0_-)\, i_L(0_-)$ 变化k倍时，零输入响应也增加k倍

注意齐次性

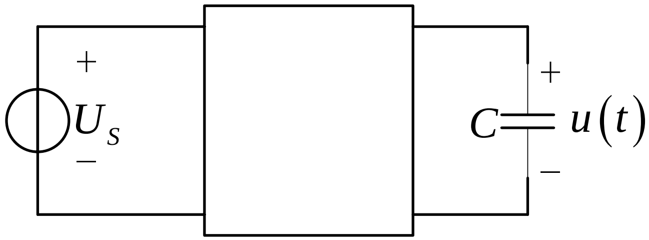
零状态响应

当输入激励变化k倍，零状态响应也增加k倍

## 6-17

已知 $u_s(t)=10\varepsilon(t)$ 时 $u_c=(3+5e^{-3t})\varepsilon(t)$

求 $u_s(t)=5\varepsilon(t)$ 时的全响应




解 $u_s(t)=10\varepsilon(t)$ $u_c=(3+5e^{-3t})\varepsilon(t)$

$$u_c(\infty)=(3+5e^{-3\times\infty})=3$$

$$u_c(0_+)=(3+5e^{-3\times 0_+})=8$$

根据零状态齐次性 $u_s(t)=5\varepsilon(t)$ $u_c(\infty)=1.5$

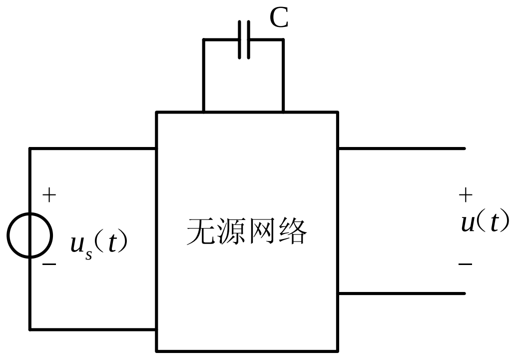
零输入

$$u_c(t)=(1.5-1.5e^{-3t})+8e^{-3t}=1.5+6.5e^{-3t}$$

零状态

电容电压电感电流的全响应方程可以分解出零输入和零状态 响应，其它不可以。

**6-17**




当 $u_s(t)=10\varepsilon(t)$ 作用时，响应 $u(t)=10+4e^{-t}\varepsilon(t)$

当 $u_s(t)=5\varepsilon(t)$ 作用时，响应 $u(t)=5+6e^{-t}\varepsilon(t)$

求零输入响应

$u(t)=u_{cx}+u_{cf}$（$u_{cx}$ → 零输入，$u_{cf}$ → 零状态）

解 $u_s(t)=10\varepsilon(t)$

$$10+4e^{-t}=u_{cx}+u_{cf}$$

根据零状态齐次性

$u_s(t)=5\varepsilon(t)$

$$5+6e^{-t}=u_{cx}+10/5\times u_{cf}$$

则

$$u_{cx}=8e^{-t}\varepsilon(t)$$

---

当 $u_s(t)=10\varepsilon(t)$ $u_c(0\_)=20V$

$u(t)=10+4e^{-t}\varepsilon(t)$

当 $u_c(0\_)=30V$ $u(t)=10+8e^{-t}\varepsilon(t)$

求零状态响应

$u(t)=u_{cx}+u_{cf}$（$u_{cx}$ → 零输入，$u_{cf}$ → 零状态）

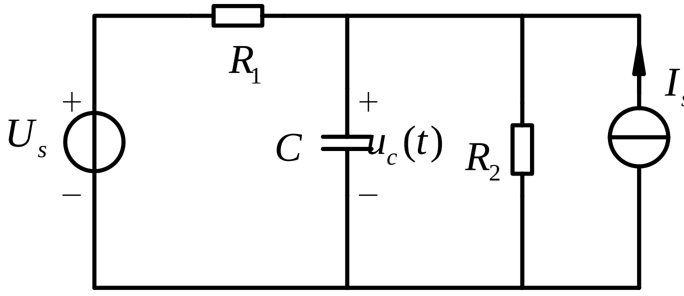
解 当 $u_c(0\_)=20V$ $10+4e^{-t}=u_{cx}+u_{cf}$

根据零输入齐次性

当 $u_c(0\_)=30V$

$$10+8e^{-t}=30/20\times u_{cx}+u_{cf}$$

$$u_{cf}=10-4e^{-t}$$




电路初始状态保持不变

$U_s=1\quad I_s=0\quad u_c(t)=0.5+2e^{-2t}$

$U_s=0\quad I_s=1\quad u_c(t)=2+0.5e^{-2t}$

求电阻和电容值。

求当 $U_s=1\quad I_s=1$电路的响应。

---

**1**）当 $U_s=1\quad I_s=0\quad u_c(t)=0.5+2e^{-2t}$

则 $u_c(\infty)=0.5\quad u_c(0_+)-u_c(\infty)=2\quad u_c(0_+)=2.5\quad U_c(\infty)=\dfrac{R_2}{R_1+R_2}U_s$

---

**2**）当 $U_s=0\quad I_s=1\quad u_c(t)=2+0.5e^{-2t}$

则 $u_c(\infty)=2\quad u_c(0_+)-u_c(\infty)=0.5\quad u_c(0_+)=2.5\quad U_c(\infty)=\dfrac{R_2R_1}{R_1+R_2}I_s$

$\tau=R_1//R_2\times C=0.5$

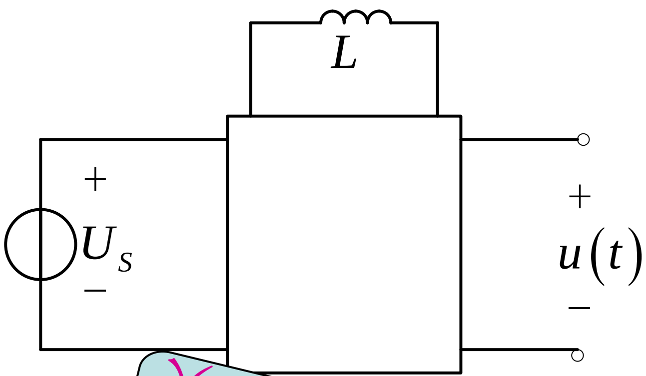
---

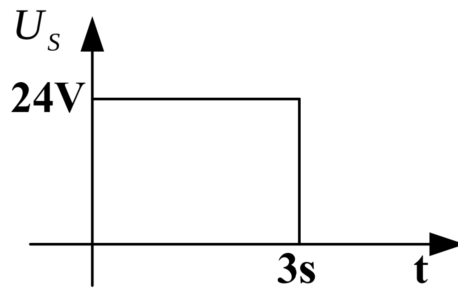
**3**）$u_c^{'}(\infty)=0.5V\quad u_c^{''}(\infty)=2V\quad u_c(\infty)=u_c^{'}(\infty)+u_c^{''}(\infty)=2.5V$

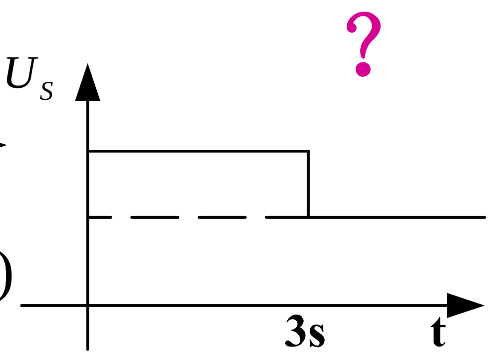
$u_c(t)=u_c(\infty)+\left[u_c(0_+)-u_c(\infty)\right]e^{-2t}=2.5V$

已知 $u_s = 8\varepsilon(t)$ 时 $u = (3 + 2e^{-3t})\varepsilon(t)$  $u_s = 16\varepsilon(t)$时 $u = (6 + 5e^{-3t})\varepsilon(t)$

求如图所示激励的全响应




$$u_s = 24\varepsilon(t) - 24\varepsilon(t-3)$$

零输入

零状态

（1）

$$3 + 2e^{-3t} = u_x + u_h$$

$$6 + 5e^{-3t} = u_x + 2u_h$$

$$u_x = 2\left(3 + 2e^{-3t}\right) - \left(6 + 5e^{-3t}\right) = -e^{-3t}$$

$$u_h = 3 + 3e^{-3t}$$

（2）$u_h = 3u_h = \left(9 + 9e^{-3t}\right)\varepsilon(t)$

零状态线性

$$u_h = \left(9 + 9e^{-3t}\right)\varepsilon(t) - \left(9 + 9e^{-3(t-3)}\right)\varepsilon(t-3)$$

$$u = u_h + u_x = \left(9 + 9e^{-3t}\right)\varepsilon(t) - \left(9 + 9e^{-3(t-3)}\right)\varepsilon(t-3) - e^{-3t}$$

## 单位冲激响应 $h(t)$

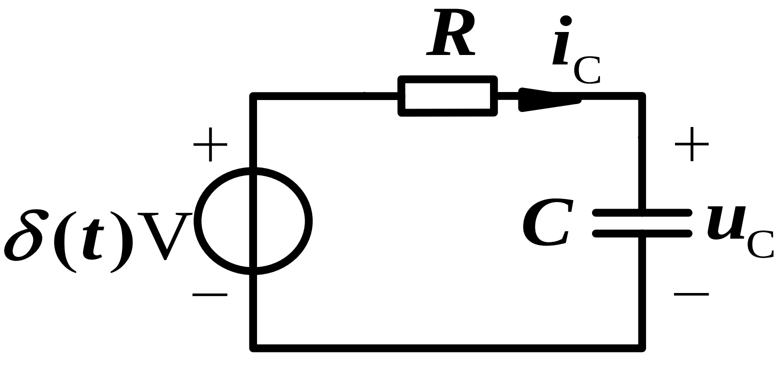
（1）定义：电路对单位冲激输入的零状态响应称为单位冲激响应，并用$h(t)$表示。

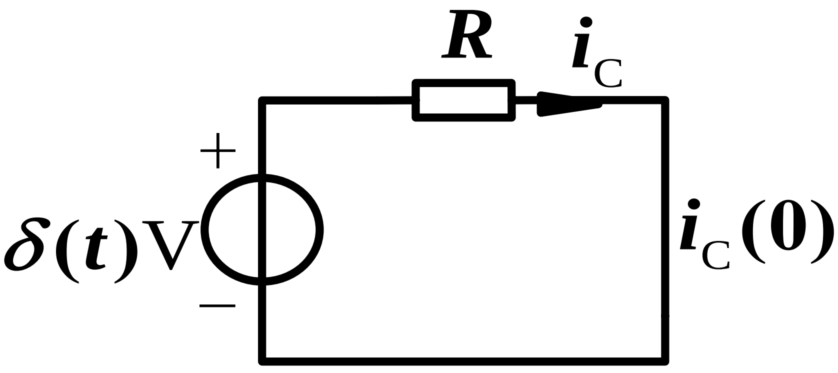
（2）求解方法

① 求在（$0_-$～ $0_+$）瞬间，冲激激励作用下，在零状态电路中建立的初始状态（因电容电流和电感电压跃变，不满足换路定则）。

② 求（t≥ $0_+$）时，初始状态作用下的零输入响应。

【例1】求冲激响应$u_C(t)$和$i_C$（$t$）。




解：分两个时间段分析求冲激响应。

（1）在$0_-\sim 0_+$间，电容充电，电容等效为短路。

$$i_C(0)=\frac{\delta(t)}{R}$$

根据电容电压电流关系

$$u_c(0_+) = u_c(0_-) + \frac{1}{C}\int_{0_-}^{0_+} i_{\mathrm{c}}(0)\mathrm{d}t = \frac{1}{C}\int_{0_-}^{0_+} \frac{\delta(t)}{R}\mathrm{d}t = \frac{1}{RC}$$

（2）在t≥$0_+$间，电容对电阻放电，电路为零输入响应。

$$\therefore \quad h_u(t) = u_{\mathrm{C}}(t) = u_{\mathrm{C}}(0_+)e^{-\frac{t}{RC}}\varepsilon(t) = \frac{1}{RC}e^{-\frac{t}{RC}}\varepsilon(t)(\mathrm{V})$$

$$\therefore \quad h_u(t) = u_C(t) = u_C(0_+)e^{-\frac{t}{RC}}\varepsilon(t) = \frac{1}{RC}e^{-\frac{t}{RC}}\varepsilon(t)$$

作为函数求导

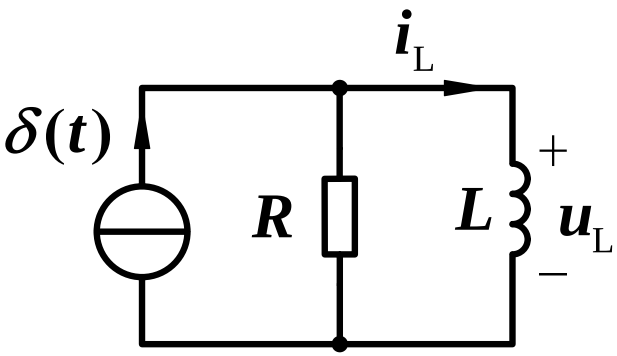
$$\therefore \quad i_c(t) = C\frac{\mathrm{d}u_C(t)}{\mathrm{d}t}$$

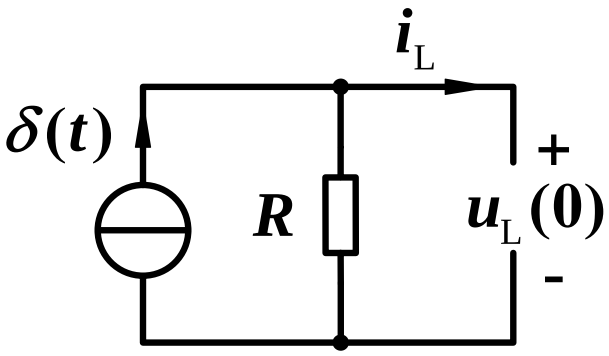
$$= C\frac{1}{RC}\left[(-\frac{1}{RC})e^{-\frac{t}{RC}}\varepsilon(t) + e^{-\frac{t}{RC}}\frac{\mathrm{d}\varepsilon(t)}{\mathrm{d}t}\right]$$

$$= -\frac{1}{R^2C}e^{-\frac{t}{RC}}\varepsilon(t) + \frac{1}{R}\left[e^{-\frac{t}{RC}}\delta(t)\right]_{t=0}$$

$$= -\frac{1}{R^2C}e^{-\frac{t}{RC}}\varepsilon(t) + \frac{1}{R}\delta(t)$$

【例2】求冲激响应$i_L$（$t$）和$u_L(t)$ 。




解：分两个时间段分析, 求冲激响应。

（1）在$0_-$～$0_+$间，电感等效为开路。

$$u_L(0)=\delta(t)R$$

（2）根据电感电压电流关系。

$$i_L(0_+) = i_L(0_-) + \frac{1}{L}\int_{0_-}^{0_+} u_L \mathrm{d}t = \frac{1}{L}\int_{0_-}^{0_+} R\delta(t)\mathrm{d}t = \frac{R}{L}$$

$$\therefore \quad i_L(0_+) = \frac{R}{L}$$

（2）在t≥$0_+$间，电容对电阻放电，电路为零输入响应。

$$\therefore \quad h_i(t) = i_L(t) = i_L(0_+)e^{-\frac{t}{RC}}\varepsilon(t) = \frac{R}{L}e^{-\frac{Rt}{L}}\varepsilon(t)$$

作为函数求导

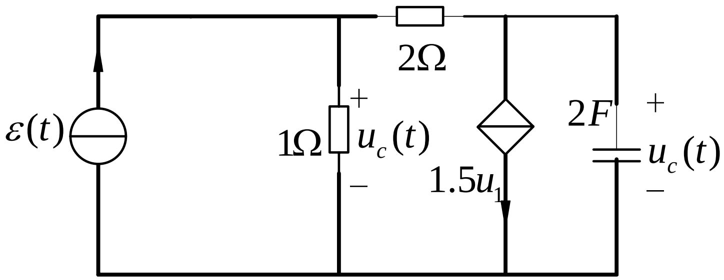
$$\therefore \quad h_u(t) = u_L(t) = L\frac{\mathrm{d}i_L(t)}{\mathrm{d}t}$$

$$= L\frac{R}{L}\left[(-\frac{R}{L})e^{-\frac{R}{L}t}\varepsilon(t) + e^{-\frac{t}{RC}}\frac{\mathrm{d}\varepsilon(t)}{\mathrm{d}t}\right]$$

$$= -\frac{R^2}{L}e^{-\frac{R}{L}t}\varepsilon(t) + R\left[e^{-\frac{R}{L}t}\delta(t)\right]_{t=0}$$

$$= -\frac{R^2}{L}e^{-\frac{R}{L}t}\varepsilon(t) + R\delta(t)$$

## 6-19 求电路单位阶跃响应

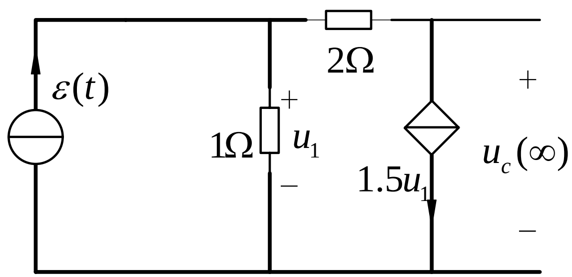



1）求初始电容电压

$$u_c(0_-)=0V \qquad u_c(0_-)=u_c(0_+)=0V$$

单位阶跃

2）求稳态值

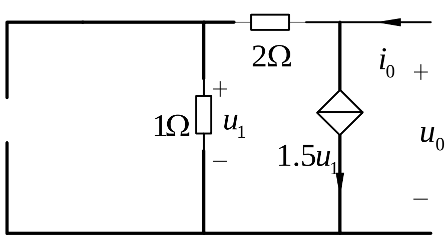



$$\varepsilon(t)=\frac{u_1}{1}+1.5u_1=1 \qquad u_1=\frac{1}{2.5}V$$

$$u_c(\infty)-u_1+2\times1.5u_1=0V$$

$$u_c(\infty)=-2u_1=-0.8V$$

3）求时间常数




$$u_0=2u_1+u_1=3u_1 \qquad i_0=1.5u_1+u_1/1=2.5u_1$$

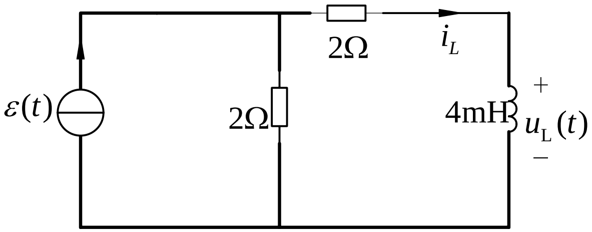
$$R=\frac{3u_1}{2.5u_1}=\frac{6}{5}\Omega \qquad \tau=R\times C=1.2\times2=2.4S$$

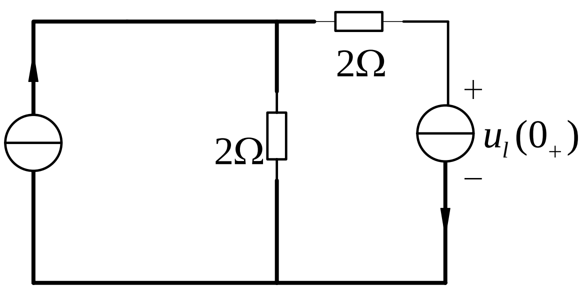
4）

$$u(t)=u(\infty)+\left[u(0_+)-u(\infty)\right]e\frac{-t}{\tau}$$

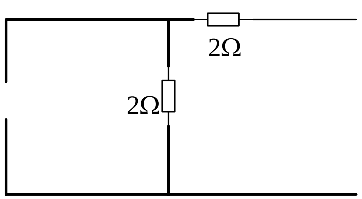
$$=\left(-0.8+0.8e^{-\frac{t}{2.4}}\right)\varepsilon(t)V$$

求电路单位阶跃响应




3）求时间常数




$$R = 4\Omega$$

$$\tau = L/R = 0.004/4 = 0.001S$$

1）求初始电流

$$i_L(0_-) = 0$$

单位阶跃

$$i_L(0_+) = i_L(0_-) = 0$$

2）求初始值

$$u_l(0_+) = 1\times 2 = 2V$$

3）求稳态值 $u_l(\infty) = 0V$

4）$u(t) = 2e^{-1000t}\varepsilon(t)V$

单位冲击为单位阶跃的导数

$$u_\delta(t) = \frac{du_\varepsilon}{dt} = \frac{d\,2e^{-1000t}\varepsilon(t)}{dt}$$

$$= 2\delta(t) - 2000e^{-1000t}\varepsilon(t)$$

## 6-20 求图示电路电感电压单位冲击响应

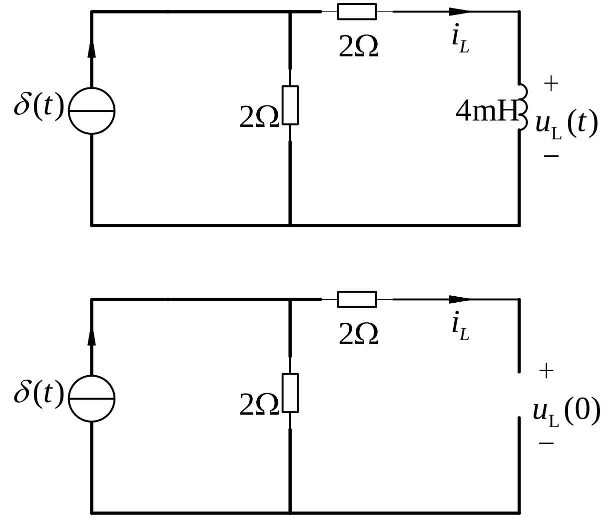



图1

电感开路，电容短路

1）根据图1求初始电压

$$u_L(0) = 2\delta(t)$$

$$i_L(0_+) = \frac{1}{L}\int_{0^-}^{0^+} u_L(0)dt = \frac{1}{L}\int_{0^-}^{0^+} 2\delta(t)dt = 500A$$

2）由图2求初始值

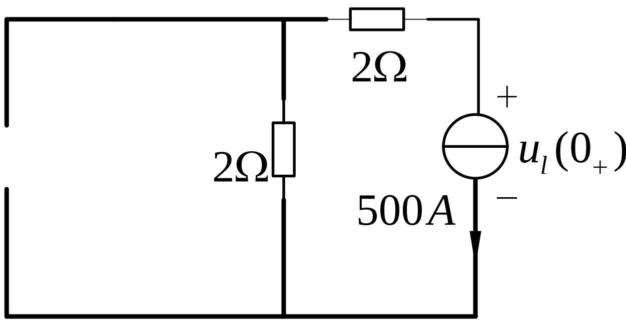



图2

$$u_l(0_+) = -500\times(2+2)$$

$$= -2000V$$

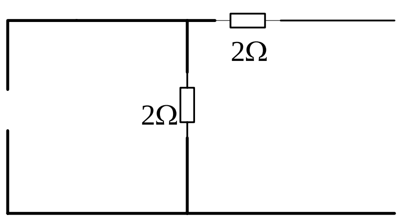
3）求稳态值

$$u_l(\infty) = 0V$$

4） $u(t) = 2\delta(t) - 2000e^{-1000t}\varepsilon(t)V$

补充电感电压在零时刻值

4）求时间常数




$$R = 4\Omega$$

$$\tau = L/R = 0.004/4 = 0.001S$$

# 假二阶电路

电路等效为两个一阶动态电路，根据三要素法分别求解。

# 四、二阶动态电路的方程及三种响应

列写电路动态方程，根据特征根的取值来判断其相应特性。

标准形式

$$\frac{d^2 y}{dt^2} + 2\alpha \frac{dy}{dt} + \omega^2 y = 0$$

特征方程

$$\lambda^2 + 2\alpha\lambda + \omega^2 = 0$$

$$\lambda_1 = -\alpha + \sqrt{\alpha^2 - \omega^2} \qquad \lambda_2 = -\alpha - \sqrt{\alpha^2 - \omega^2}$$

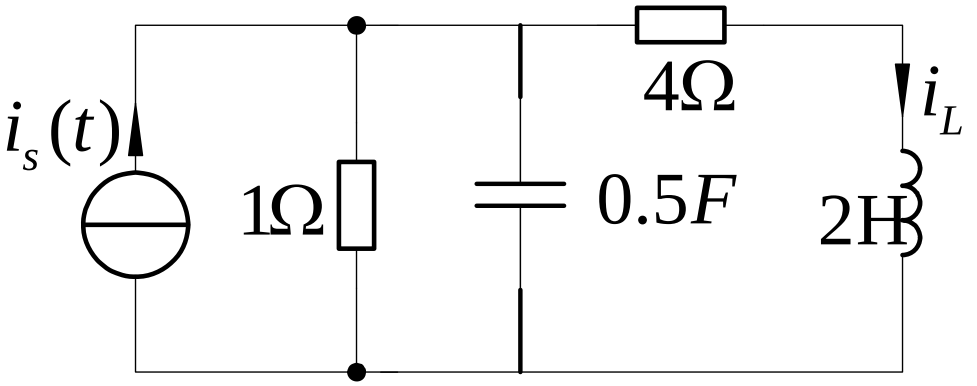
两个不相等的实数根，过阻尼； $\alpha > \omega$

两个相等的实数根，临界阻尼； $\alpha = \omega$

两个不相等的复数根，欠阻尼； $\alpha < \omega$

两个共轭纯虚数根，无阻尼； $\alpha = 0$

列写以电感电流为输出变量的输入输出方程。




$$a\lambda^2 + b\lambda + c = 0$$

$$\lambda_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{di_L^2}{dt^2} + 4\frac{di_L}{dt} + 5i_L = i_s \qquad \lambda^2 + 4\lambda + 5 = 0$$

激励为零

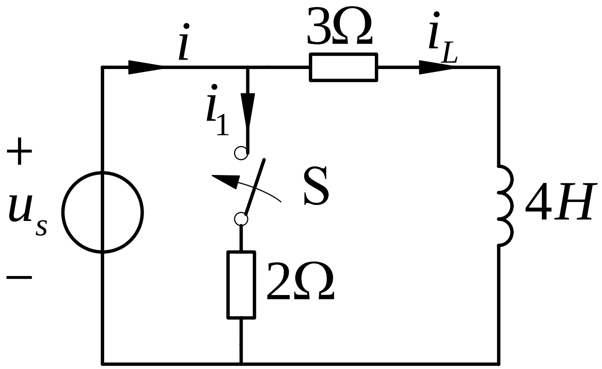
$$\lambda^2 + 2\alpha\lambda + \omega^2 = 0 \qquad \alpha = 2 \qquad \omega = \sqrt{5}$$

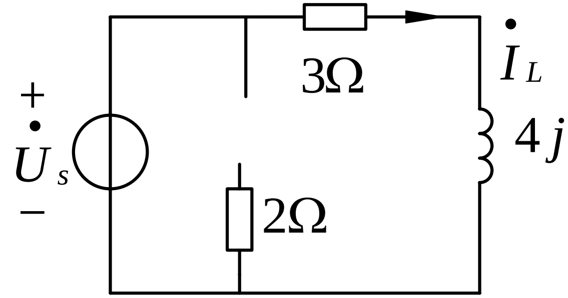
$\alpha < \omega$　两个不相等的复数根，欠阻尼；

# 第六章 线性动态电路的时域分析

## 非直流量激励下的暂态响应分析

**求换路后的各电流初始值** $u_s = 10\cos(t)V$



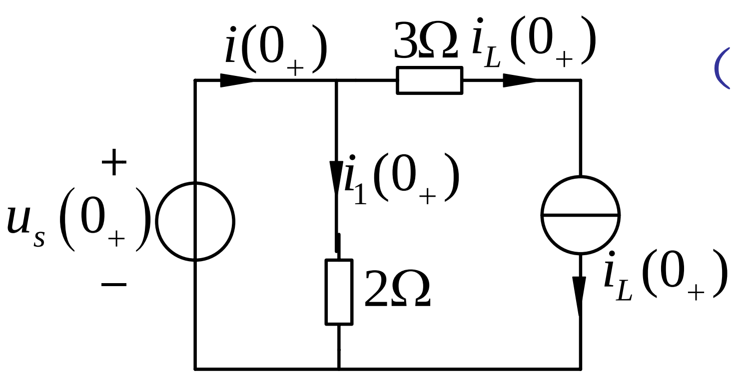




**（1）求换路前稳态值**

$$\dot{I}_{Lm} = \frac{\dot{U}_{sm}}{Z} = \frac{10\angle 0^0}{5\angle 53^0} = 2\angle -53^0$$

$$i_L = 2\cos(t - 53^0)$$

$$i_L(0_+) = i_L(0_-) = 2\cos(0_- - 53^0) = 1.2$$

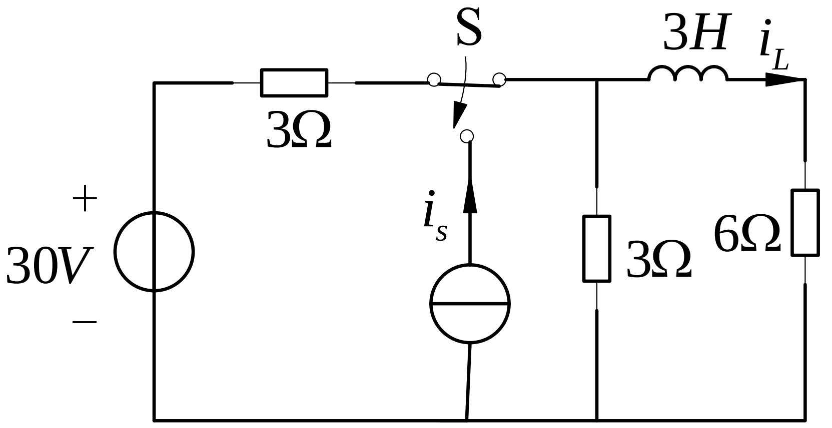



**（2）求初始值**

$$u_s(0_+) = 10\cos(0_+) = 10 \qquad i_1(0_+) = \frac{u_s(0_+)}{2} = \frac{10}{2} = 5$$

$$i(0_+) = i_L(0_+) + i_1(0_+) = 5 + 1.2 = 6.2A$$

# 求换路后的电感电流 $i_s = 9\sqrt{2}\cos 3t$

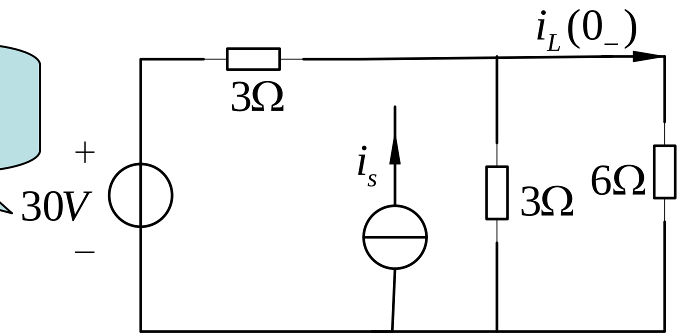



## （1）求换路前稳态值

$$i_L(0_-) = \frac{30}{3+6//3} \times \frac{3}{3+6} = 2A$$

$$i_L(0_+) = i_L(0_-) = 2A$$

换路前稳态电路

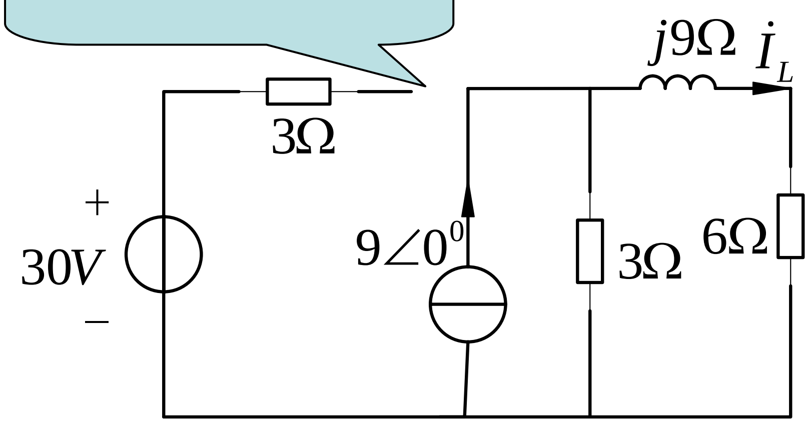



## （2）求换路后稳态值（相量分析法）

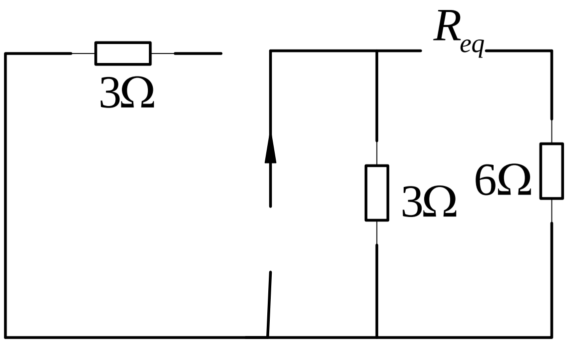
换路后稳态电路

$$\dot{I}_L = \frac{3}{6+9j+3} 9\angle 0^0 = 1.5\sqrt{2}\angle -45^0$$

$$i_L = 3\cos\left(3t - 45^0\right)$$



## （3）求时间常数




$$R_{eq} = 9\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{1}{3}s$$

## （4）代入三要素公式

稳态值在零正时刻的值

$$i_L(t) = i_L(\infty) + \left[ i_L(0_+) - i_L(\infty)\big|(t = 0_+) \right] e^{\frac{-t}{\tau}}$$

$$i_L(t) = 3\cos\left(3t - 45^0\right) + \left[ 2 - 3\cos\left(3 \times 0_+ - 45^0\right) \right] e^{-3t}$$

$$= 3\cos\left(3t - 45^0\right) + \left[ 2 - 3\cos\left(3 \times 0_+ - 45^0\right) \right] e^{-3t}$$

$$= 3\cos\left(3t - 45^0\right) - 0.121 e^{-3t}$$

# 答疑时间及地点

地点：教**11A420**

**11**月**27** 日全天

**11**月**28** 日下午全天

考试时间**11**月**29**日（周六）