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Robust design of fixture configuration

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Abstract

The paper deals with robust design of fixture configuration. It aims to investigate how fixture element deviations and machine tool volumetric errors affect machining operations quality. The locator position configuration is then designed to minimize the deviation of machined features with respect to the applied geometric tolerances.
The proposed approach represents a design step that goes further the deterministic positioning of the part based on the screw theory, and may be used to look for simple and general rules easily applicable in an industrial context.
The methodology is illustrated and validated using simulation and simple industrial case studies.

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Keywords: Tolerancing; Error; Modular Fixture.

1. Introduction

When a workpiece is fixtured for a machining or inspection operation, the accuracy of an operation is mainly determined by the efficiency of the fixturing method. In general, the machined feature may have geometric errors in terms of its form and position in relation to the workpiece datum reference frame. If there exists a misalignment error between the workpiece datum reference frame and machine tool reference frame, this is known as localization error [1] or datum establishment error [2]. A localization error is essentially caused by a deviation in the position of the contact point between a locator and the workpiece surface from its nominal specification. In this paper, such a theoretical point of contact is referred to as a fixel point or fixel, and its positioning deviation from its nominal position is called fixel error. Within the framework of rigid body analysis, fixel errors have a direct effect on the localization error as defined by the kinematics between the workpiece feature surfaces and the fixels through their contact constraint relationships [3].

The localization error is highly dependent on the configuration of the locators in terms of their positions relative to the workpiece. A proper design of the locator configuration (or locator layout) may have a significant impact on reducing the localization error. This is often referred to as fixture layout optimization [4].

A main purpose of this work is to investigate how geometric errors of a machined surface (or manufacturing errors) are related to main sources of fixel errors. A mathematic framework is presented for an analysis of the relationships among the manufacturing errors, the machine tool volumetric error, and the fixel errors. Further, optimal fixture layout design is specified as a process of minimizing the manufacturing errors. This paper goes beyond the state of the art, because it considers the volumetric error in tolerancing. Although the literature demonstrates that the simple static volumetric error considered here is only a small portion of the total volumetric error, a general framework for the inclusion of volumetric error in tolerancing is established.

There are several formal methods for fixture analysis based on classical screw theory [5,6] or geometric perturbation techniques [3]. In nineties many studies have been devoted to model the part deviation due to fixture [7]. Sodenberg calculated a stability index to evaluate the goodness of the locating scheme [8]. The small displacement torsor concept is used to model the part deviation due to geometric variation of the part-holder [9]. Conventional and computer-aided fixture design procedures have been described in traditional design manuals [10] and recent literature [11,12], especially for designing modular fixtures [13]. A number of methods for localization error analysis and reduction have been reported. A mathematical representation of the localization error was given in [14] using the concept of a displacements screw vector. Optimization techniques were sug-

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gested to minimize the magnitude of the localization error vector or the geometric variation of a critical feature [14,15]. An analysis is described by Chouduri and De Meter [2] to relate the locator shape errors to the worst case geometric errors in machined features. Geometric deviations of the workpiece datum surfaces were also analyzed by Chouduri and De Meter [2] for positional, profile, and angular manufacturing tolerance cases. Their effects on machined features, such as by drilling and milling, were illustrated. A second order analysis of the localization error is presented by Carlson [16]. The computational difficulties of fixture layout design have been studied with an objective to reduce an overall measure of the localization error for general three dimensional (3D) workpieces such as turbine air foils [1,4]. A more recent paper shows a robust fixture layout approach as a multi-objective problem that is solved by means of Genetic Algorithms [17]. It considers a prismatic and rigid workpiece, the contact between fixture and workpiece is without friction, and the machine tool volumetric error is not considered.

About the modeling of the volumetric error, several models have been proposed in literature. Ferreira *et al.* [18,19] have proposed quadratic model to model the volumetric error of machines, in which each axis is considered separately, thoghether with a methodology for the evaluation of the model parameters. Kiridena and Ferreira in a series of three papers [20–22] discuss how to compensate the volumetric error can be modeled, the parameters of the model evaluated, and then the error compensated based on the model and its parameters, for a three-axis machine. Dorndorf *et al.* [23] describe how volumetric error models can help in the error budgeting of machine tools. Finally, Smith *et al.* [24] describe the application of volumetric error compensation in the case of large monolithic part manufacture, which poses serious difficulties to traditional volumetric error compensation. Anyway, it is worth noting that all these approaches are aimed at volumetric error compensation: generally volumetric error is not considered for simulation in tolerancing.

In previous papers a statistical method to estimate the position deviation of a hole due to the inaccuracy of all the six locators of the 3-2-1 locating scheme was developed for 2D plates and 3D parts [25,26]. In the following, a methodology for robust design of fixture configuration is presented. It aims to investigate how fixel errors and machine tool volumetric error affect machining operations quality. In §2 the theoretical approach is introduced, in §3 a simple industrial case study is presented, and in §4 some simple and general rules easily applicable in an industrial contest are discussed.

2. Methodology for the simulation of the drilling accuracy

To illustrate the proposed methodology, the case study of a drilled hole will be considered. The case study is shown in Fig. 1. A location tolerance specifies the hole position. Three locators on the primary datum, two on the secondary datum, and one on the tertiary determine the position of the workpiece. Each locator has coordinates related to the machine tool reference frame, represented by the following six terns of values:

$$\begin{matrix} \mathbf{p}_1\left(x_1, y_1, z_1\right) & \mathbf{p}_2\left(x_2, y_2, z_2\right) & \mathbf{p}_3\left(x_3, y_3, z_3\right) \\ \mathbf{p}_4\left(x_4, y_4, z_4\right) & \mathbf{p}_5\left(x_5, y_5, z_5\right) & \mathbf{p}_6\left(x_6, y_6, z_6\right) \end{matrix} \tag{1}$$

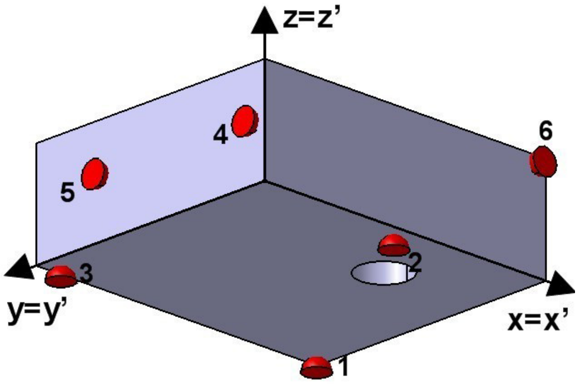

Fig. 1. Locator configuration schema.

The proposed approach considers the uncertainty source in the positioning error of the machined hole due to the error in the positioning of the locators, and the volumetric error of the machine tool. The final aim of the model is to define the actual coordinates of the hole in the workpiece reference system. The model input includes the nominal locator configuration, the nominal hole location (supposed coincident with the drill tip) and direction (supposed coincident with the drill direction), and the characteristics of typical errors which can affect this nominal parameters.

2.1. Effect of locator errors

The positions of the six locators are completely defined by their eighteen coordinates. It is assumed that each of these coordinates is affected by an error behaving independently, according to a Gaussian $N\left(0, \sigma^2\right)$ distribution.

The actual locator coordinates will then identify the workpiece reference frame. In particular, the z' axis is constituted by the straight line perpendicular to the plane passing through the actual positions of locators $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_2$, the x' axis is the straight line perpendicular to the z' axis and to the straight line passing through the actual position of locators $\mathbf{p}_4$ and $\mathbf{p}_5$, and finally the y' axis is straightforward computed as perpendicular to both z' and x' axes. The origin of the reference frame can be obtained as intersection of the three planes having as normals the x', y', and z' axes and passing through locators $\mathbf{p}_4$, $\mathbf{p}_6$ and $\mathbf{p}_1$ respectively. The formulas for computing the axis-direction vectors and origin coordinates from the actual locators coordinates are omitted here, for reference see the work by Armillotta *et al.* [26].

The axis-direction vectors and origin coordinates define an homogeneous transformation matrix ${}^0\mathbf{R}_p$ [27], which allows to convert the drill tip coordinate expressed in the machine tool reference frame $\mathbf{P}_0$ to the same coordinates expressed in the workpiece reference frame $\mathbf{P}'_0$, through the formula:

$$\mathbf{P}'_0 = {}^0\mathbf{R}_p^{-1}\mathbf{P}_0 \tag{2}$$

2.2. Effect of machine tool volumetric error

To simulate the hole location deviation due to the drilling operation, i.e. to the volumetric error of the machine tool, the classical model of three-axis machine tool has been considered [27]. It will be assumed the drilling tool axis is coincident with the machine tool z axis, so that, in nominal conditions and at the beginning of the drilling operation, its tip position can be defined by the nominal hole location and the homogeneous vector $\mathbf{k} = [0 \quad 0 \quad 1 \quad 0]^{\mathrm{T}}$. The aim is to identify the position error $\mathbf{\Delta}_p$ of the drill tip in the machine tool reference system, and the direction error $\mathbf{\Delta}_d$ of the tool axis. According to the three-axis machine tool model it is possible to state that:

$$\mathbf{\Delta}_p = {}^0\mathbf{R}_1 {}^1\mathbf{R}_2 {}^2\mathbf{R}_3 \mathbf{P}_3 - \mathbf{P}_0 \tag{3}$$

where $\mathbf{P}_0 = [x \quad y \quad z-l \quad 1]^{\mathrm{T}}$ is the nominal drill tip location in the machine tool reference system (x, y, and z being the translations along the machine tool axes, and l being the drill length), $\mathbf{P}_3 = [0 \quad 0 \quad -l \quad 1]^{\mathrm{T}}$ is the drill tip position in the third (z axis) reference system, and ${}^0\mathbf{R}_1$, ${}^1\mathbf{R}_2$, ${}^2\mathbf{R}_3$ are respectively the perturbed transformation matrices due to the perturbed translation along the x, y, and z axes. These matrices share a similar form, for example:

$${}^0\mathbf{R}_1 = \begin{bmatrix} 1 & -\varepsilon_z(x) & \varepsilon_y(x) & x + \delta_x(x) \\ \varepsilon_z(x) & 1 & -\varepsilon_x(x) & \delta_y(x) \\ -\varepsilon_y(x) & \varepsilon_x(x) & 1 & \delta_z(x) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

where the δ and ε terms are the translation and rotation errors along and around the x, y, and z axes (e.g. $\varepsilon_z(x)$ is the rotation error around the z axis due to a translation along the x axis). Considering three transformation matrices, there are eighteen error terms. These errors are usually a function of the volumetric position (i.e. the translations along the three axes), but if the volumetric error is compensated, their systematic component can be neglected and they can be assumed to be purely random with mean equal to zero. Developing Eq. (3) leads to very complex equations; for example,

$$\begin{aligned} \Delta_{dx} = & \delta_x(x) + \delta_x(y) - \varepsilon_z(x)(\delta_y(y) + y) - \delta_y(z) \cdot \\ & \cdot (\varepsilon_z(x) + \varepsilon_z(y) - \varepsilon_x(y)\varepsilon_y(x)) + \delta_z(y)\varepsilon_y(x) - \\ & - \delta_x(z)(\varepsilon_y(x)\varepsilon_y(y) + \varepsilon_z(x)\varepsilon_z(y) - 1) - \\ & - l(\varepsilon_y(x) + \varepsilon_y(y) + \varepsilon_x(z)(\varepsilon_z(x) + \varepsilon_z(y) - \\ & - \varepsilon_x(y)\varepsilon_y(x)) + \varepsilon_x(y)\varepsilon_z(x) - \varepsilon_y(z)(\varepsilon_y(x)\varepsilon_y(y) + \\ & + \varepsilon_z(x)\varepsilon_z(y) - 1)) + (\delta_z(z) + z)(\varepsilon_y(x) + \varepsilon_y(y) + \\ & + \varepsilon_x(y)\varepsilon_z(x)) \end{aligned} \tag{5}$$

However, volumetric errors in general should be far smaller than translations along the axes, so only the first order components of Eq. (3) are usually significant. Finally, Assuming the drilling tool axis coincide with the z axis, Eq. (3) can also calculate the direction error $\mathbf{\Delta}_d$ by substituting $\mathbf{P}_0 = \mathbf{P}_3 = \mathbf{k}$.

If only the first order components are considered, it is possible to demonstrate that $\mathbf{\Delta}_p$ and $\mathbf{\Delta}_d$ are linear combination of the δ and ε terms. In particular, let's define $\mathbf{\Delta}$ as

$$\mathbf{\Delta} = \begin{bmatrix} \mathbf{\Delta}_p \\ \mathbf{\Delta}_d \end{bmatrix} \tag{6}$$

the six-elements vector containing $\mathbf{\Delta}_p$ and $\mathbf{\Delta}_d$ staked. Applying Eq. (3), neglecting terms above the second order, it is possible to demonstrate that (please note that, due format constraints, in Eq. (7) the dots ... indicate that a row of the matrix is broken over more lines, so the overall linear combination matrix appearing here is a 6 X 18 matrix)

$$\mathbf{\Delta} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0\dots \\ 0 & 0 & 0 & 0 & 0 & 0\dots \\ z-l & z-l & -l & -y & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1\dots \\ 0 & 0 & 0 & l-z & l-z & l\dots \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0\dots \\ 1 & 1 & 1 & 0 & 0 & 0\dots \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0\dots \\ 0 & 0 & 0 & 1 & 0 & 0\dots \\ -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0\dots \\ 0 & 0 & 0 & 1 & 1 & 1\dots \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0\dots \\ 0 & 0 & 0 & 0 & 0 & 0\dots \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \delta_x(x) \\ \delta_x(y) \\ \delta_x(z) \\ \delta_y(x) \\ \delta_y(y) \\ \delta_y(z) \\ \delta_z(x) \\ \delta_z(y) \\ \delta_z(z) \\ \varepsilon_x(x) \\ \varepsilon_x(y) \\ \varepsilon_x(z) \\ \varepsilon_y(x) \\ \varepsilon_y(y) \\ \varepsilon_y(z) \\ \varepsilon_z(x) \\ \varepsilon_z(y) \\ \varepsilon_z(z) \end{bmatrix} = \mathbf{Cd} \tag{7}$$

Now, let's assume that each term δ is independently distributed according to a Gaussian $N\left(0, \sigma_p^2\right)$ distribution, and that each term ε is independently distributed according to a $N\left(0, \sigma_d^2\right)$ distribution. It is then possible to demonstrate [28] that $\mathbf{\Delta}$ follows a multivariate Gaussian distribution, with null expected value and covariance matrix which can be calculated by the formula $\mathbf{C\Sigma C}^{\mathrm{T}}$, where $\mathbf{\Sigma}$ is the covariance matrix of $\mathbf{d}$, which happens to be a diagonal 18 X 18 matrix with the first nine diagonal elements equal to σ_p^2, an the remaining diagonal elements equal to σ_d^2. The final covariance matrix of $\boldsymbol{\Delta}$ is:

$$\begin{bmatrix} \begin{matrix}\sigma_d^2 y^2+\\ +2\sigma_d^2\cdot\\ \cdot(l-z)^2+\\ +3\sigma_p^2+\\ +l^2\sigma_d^2\end{matrix} & 0 & 0 & \begin{matrix}\sigma_d^2\cdot\\ \cdot(3l-\\ -2z)\end{matrix} & \begin{matrix}-\sigma_d^2\cdot\\ \cdot(l-z)\end{matrix} & 0 \\ 0 & \begin{matrix}2\sigma_d^2\cdot\\ \cdot(l-z)^2+\\ +3\sigma_p^2+\\ +l^2\sigma_d^2\end{matrix} & 0 & \begin{matrix}\sigma_d^2\cdot\\ \cdot(l-z)\end{matrix} & \begin{matrix}\sigma_d^2\cdot\\ \cdot(3l-\\ -2z)\end{matrix} & 0 \\ 0 & 0 & 3\sigma_p^2 & 0 & 0 & 0 \\ \begin{matrix}\sigma_d^2\cdot\\ \cdot(3l-\\ -2z)\end{matrix} & \begin{matrix}\sigma_d^2\cdot\\ \cdot(l-z)\end{matrix} & 0 & 4\sigma_d^2 & 0 & 0 \\ \begin{matrix}-\sigma_d^2\cdot\\ \cdot(l-z)\end{matrix} & \begin{matrix}\sigma_d^2\cdot\\ \cdot(3l-\\ -2z)\end{matrix} & 0 & 0 & 4\sigma_d^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_d^2 \end{bmatrix} \tag{8}$$

This model can be adopted to simulate the error in the location and direction of the hole due to the machine tool volumetric error.

2.3. Actual location of the manufactured hole

Now, it is possible to simulate the tip location and direction according to the model described in §2.2, and to transform it into the workpiece reference frame as described in §2.1:

$$\begin{aligned} \mathbf{P}_0{}' &= {}^0\mathbf{R}_p^{-1}\left(\mathbf{P}_0 + \boldsymbol{\Delta}_p\right) \\ \mathbf{k}' &= {}^0\mathbf{R}_p^{-1}\left(\mathbf{k} + \boldsymbol{\Delta}_d\right) \end{aligned} \tag{9}$$

With this information it is possible to determine the entrance and exit location of the hole in the workpiece reference system. Point $\mathbf{P}_0'$ and vector $\mathbf{k}'$ define a straight line, which is nothing else than the hole axis, as:

$$\begin{aligned} &\mathbf{p}' = \mathbf{P}_0{}' + s\mathbf{k}' \\ &\begin{bmatrix} p_x{}' \\ p_y{}' \\ p_z{}' \end{bmatrix} = \begin{bmatrix} P_{0x}{}' \\ P_{0y}{}' \\ P_{0z}{}' \end{bmatrix} + s\begin{bmatrix} k_x{}' \\ k_y{}' \\ k_z{}' \end{bmatrix} \end{aligned} \tag{10}$$

where $\mathbf{p}$ is a generic point belonging to the line and $s \in \mathbb{R}$ is a parameter. Defining T as the plate thickness, it is possible to calculate the values of s for which p_z' is equal respectively to 0 and T:

$$\begin{aligned} s_{exit} &= -P_{0z}{}'/k_z{}' \\ s_{entrance} &= -\left(P_{0z}{}' - T\right)/k_z{}' \end{aligned} \tag{11}$$

These values of s substituted in Eq. (10) yield respectively the coordinates of the exit and entrance point of the hole.

Finally, it is possible to calculate the distances between the two exit and entrance points of the drilled and nominal holes:

$$\begin{aligned} d_1 &= \left|\mathbf{P}_0{}' + s_{entrance}\mathbf{k}' - \mathbf{P}_0\right| \\ d_2 &= \left|\mathbf{P}_0{}' + s_{exit}\mathbf{k}' - \mathbf{P}_{0,exit}\right| \end{aligned} \tag{12}$$

where $\mathbf{P}_{0,exit}$ is the nominal location of the hole exit point. The axis of the drilled hole will be inside location tolerance zone of the hole if both the distances calculated by Eq. (12) will be lower than the half of the location tolerance value t:

$$\begin{aligned} d_1 &\leq t/2 \\ d_2 &\leq t/2 \end{aligned} \tag{13}$$

3. Case study results

The model proposed so far has been considered to identify the expected quality due to locator configuration, given a machine tool volumetric error. To identify which is the optimal one an experiment has been designed and results have been analyzed by means of analysis of variance (ANOVA) [29].

Because the aim of the research regards only the choice of locators' positions, most of the model parameters can be kept constant. The constant parameters include: the nominal size of the plate (100 x 120 x 60 mm); the standard deviation of the random errors in locator positioning ($\sigma = 0.01$ mm); the nominal location of the entrance ($\mathbf{P}_0 = [40 \quad 70 \quad 60]^{\mathrm{T}}$) and exit ($\mathbf{P}_0 = [40 \quad 70 \quad 0]^{\mathrm{T}}$) points of the hole; the length of the drill ($l = 60$ mm); the standard deviation of the machine tool axes positioning errors ($\sigma_p = 0.01$ mm) and of their rotational errors ($\sigma_d = 0.01$); the location tolerance value ($t = 0.1$ mm); the plate thickness ($T = 60$ mm). Each locator has instead been left free to change in order to evaluate its influence on the drilling accuracy; candidate configurations will be introduced in the next paragraphs, together with their impact discussion.

By substituting the simulation parameters indicated so far in Eq. (8) the following covariance matrix is yielded (all values are in [mm^2]):

$$\begin{pmatrix} 68 & 0 & 0 & -0.18 & 0.18 & 0 \\ 0 & 63 & 0 & -0.18 & -0.18 & 0 \\ 0 & 0 & 30 & 0 & 0 & 0 \\ -0.18 & -0.18 & 0 & 0.01 & 0 & 0 \\ 0.18 & -0.18 & 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.003 \end{pmatrix} \cdot 10^{-5} \tag{14}$$

The considered performance indicator is the fraction of conforming parts generated by a specific locator configuration, i.e. the fraction of parts for which both of the inequalities in Eq. (13) hold. The conforming fraction has been evaluated ten times for each experimental condition, for each evaluation ten thousand workpieces have been simulated. Of course, higher values of this performance indicator are preferable.

The ANOVA analysis has worked efficiently, with its hypotheses correctly verified. The main effect plot in Fig. 2 sum-

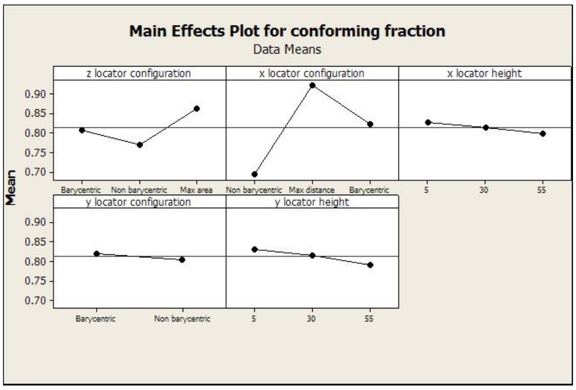

Fig. 2. Main effect plot for the fraction of conforming workpieces.

marizes the results, which are described in depth in the following paragraphs.

3.1. *Impact of* $\mathbf{p}_1$, $\mathbf{p}_2$ *and* $\mathbf{p}_3$ *locator configuration*

The $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$ locators define the part z' axis, so they have been indicated in Fig. 2 as "z locator configuration". To evaluate their impact on the hole accuracy three candidate configurations have been considered. The first one ("max area") tries to cover as much as possible the surface of the workpiece touched by the locators themselves. The second one ("barycentric") has the barycenter of the locators coincident with the hole position, but with an area coverage smaller than the max area configuration. The last one ("non barycentric") has the same area coverage of the barycentric one, but is far from the hole. Please note that the plate equilibrium has been neglected in this first analysis.

The ANOVA suggests that the best condition is the one in which the area coverage is maximum, and that given the same area coverage, having a barycentric distribution is preferable. The impact of the z locator configuration is very relevant, changing the conforming fraction of about 10%.

3.2. *Impact of* $\mathbf{p}_4$ *and* $\mathbf{p}_5$ *locator configuration*

The $\mathbf{p}_4$ and $\mathbf{p}_5$ locators define the part x' axis. Two factors have been considered for them: their height with the hypothesis that they have the same one ("x locator height" in Fig. 2), and their positions in the y direction ("x locator configuration").

Three candidate heights have been considered, 5, 30 and 55 mm. It seems that it is slightly better to have the locators placed at the lower height, but the impact is quite small (about 1%).

The impact of the x locator configuration, accounting for about the 20% of the conforming fraction, is far more relevant. Similarly to the z locator configuration, three configurations have been considered. The first one ("max distance") maximizes the distance between the two locators. The second one ("barycentric") has the barycenter of the two locators in correspondence of the hole axis, but with a distance smaller than the "max distance" one. The last one ("non barycentric") has the same distance of the barycentric, but it has the barycenter of the two locators far from the hole axis. The best condition is the one with the maximum distance between the two locators, followed by the barycentric one.

3.3. *Impact of* $\mathbf{p}_6$ *locator configuration*

The $\mathbf{p}_6$ locator has a smaller impact with respect to the other ones. Similarly to the x locator configuration, it has been investigated for its height ("y locator height") and position ("y locator configuration"). The results are similar to the x locator configuration impact, so the height should be kept at minimum, and the locator should be barycentric, i.e. it should correspond to the hole axis. However, its impact accounts for only about 1% of the conforming fraction.

3.4. *Impact of other factors*

The ANOVA has shown that the interactions between the factors are relevant, too, i.e. the positions of all of the six locators collaborate together to define the accuracy. However, the impact of the interaction factors is one order of magnitude smaller than the direct impact of the factors, so in general it is advisable not to consider them in the planning of the optimal locator configuration.

4. Conclusions

This work proposed a methodology for robust design of fixture configuration considering the random error of locators positions (due to the locator mounting on the machine table, the contact on irregular surfaces of the workpiece, etc.) and the volumetric error of the machine tool adopted for the operation.

The simple industrial case study of drilling operation has been considered. However, some simple and general rules can be drawn by the experimental analysis:

1. The three-point datum locators (the three locators of the z locator configuration) should cover as much as possible the surface they are in contact with. If more configurations share similar surface coverage, it is advisable that the locators barycenter is as near as possible to the one of the machined feature (the hole in the case study).
2. The two-point datum locators (the two locators of the x locator configuration) should have the maximum distance from each other. If more distributions share similar distances, the configuration with locators barycenter the most near to the one of the machined feature (the hole in the case study) should be selected.
3. The single locator of the third datum (the one of the y locator configuration) should be located in correspondence of the barycenter of the machined feature (the hole in the case study).
4. Even if the impact of the height of the locators $\mathbf{p}_4$, $\mathbf{p}_5$ and $\mathbf{p}_6$ is small, it is advisable they are located at the minimum height.

Future developments will be aimed to the extension of the methodology to other, more complex geometric features and tolerances.

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