SELF-RECONFIGURATION OF UNDERACTUATED REDUNDANT MANIPULATORS WITH OPTIMIZING THE FLEXIBILITY ELLIPSOID*

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Abstract: The multi-modes feature, the measure of the manipulating flexibility, and self-reconfiguration control method of the underactuated redundant manipulators are investigated based on the optimizing technology. The relationship between the configuration of the joint space and the manipulating flexibility of the underactuated redundant manipulator is analyzed, a new measure of manipulating *flexibility ellipsoid for the underactuated redundant manipulator with passive joints in locked mode is* proposed, which can be used to get the optimal configuration for the realization of the self-reconfiguration control. Furthermore, a time-varying nonlinear control method based on harmonic inputs is suggested for fulfilling the self-reconfiguration. A simulation example of a three-DOFs underactuated manipulator with one passive joint features some aspects of the investigations.

Key words: Underactuated manipulators Self-reconfiguration Optimization Nonlinear control

0 INTRODUCTION

Underactuated mechanism and manipulator can be used in some fields such as space technology, cooperation robot and metamorphic mechanism. In the space field, for the sake of no losing the useful function or realizing the reconfiguration of the system, the fault tolerance based on the underactuated technology is essential when some actuated components reveal some troubles [1]. An underactuated manipulator also can be designed as a cooperation robot that is to say COBOT [2]. The drivers of the COBOT are not for actuating the machine but for providing a kinematics constraint that is usually nonholonomic. The COBOT needs outside force that is provided by operator and can complete some accurate applications such as in biology engineering, surgical, and semiconductor manufacture and so on. In the field of mechanisms, the metamorphic mechanism has multi-modes and can be transformed from one mode to another, had been presented recently[3]. The transformation between the different modes is likely to result in change of the DOFs or constraints of the metamorphic mechanism. It is obviously that control the underactuated, redundant actuated and flexible mechanism cannot be evaded. Therefore, the underactuated system becomes an attractive research field gradually.

The researches on the underactuated manipulators manifest the system cannot be controlled by kinematics. The motion of the passive joint can be moved by the dynamic coupling only [4,5]. Jain, et al [6] have shown that the dynamic coupling is second order nonholonomic constraints of the underactuated manipulator. In contrast with the fact that nonholonomic system is studied extensively about one hundred of years in mechanics, but the motion planning and control of the nonholonomic system is no longer than two decade, and the studies limited in the first order nonholonomic system (such as wheel moving robot[7], hopping robot[8] and space robot [9]) mainly. In the aspect of the research on the underactuated manipulators, Anthoney, et al[10] have investigated the stability of the motion, Arai, et al [11] have proposed a time-scaling method and achieved the position control of the system. Lee, et al [12] *have presented several kinds of nonlinear control method for un*deractuated acrobot. These researches on the control of underactuated system have revealed that these methods are nothing but nonlinear, time varying, and discrete in nature. The fact is that Brockett [13] have proved that there is no smooth and static state feedback law that stabilizes the system to a given configuration asymptotically. An obvious feature of the nonholonomic system is that it is controllable in a configuration space with more dimensions than that of the input space. So that much attentions have been paid on the control of the nonholonomic system.

The underactuated mechanism or manipulator also is disobedient to the fundamental principle of the mechanism designing theory that is the number of the actuated components should equal to that of the DOFs of the mechanism. The underactuated manipulator had been suggested firstly is not by reason that it has some merits distinctly, but some researches show that the underactuated mechanism designed purposely also is valuable. For example, Rivhter, et al [14] have measured multi-dimensions force by a flexible underactuated manipulator, Nakamura, et al[15] have designed a nonholonomic manipulator based on the wheel rolling contact and controlled a four DOFs planar manipulator by two-inputs. He, et al [16] have proposed a collision-free motion plan algorithm for the underactuated redundant manipulator. Based on the results that have been discussed above, we can conclude that the investigation about the underactuated manipulator deliberately may result in some new phenomenon to be discovered, techniques to be proposed or theory to be formed, therefore, could be develop the mechanics potentially.

In this paper, we explore the static feature and self-reconfiguration control method for the underactuated redundant manipulators.

1 FLEXIBILITY ELLIPSOID

The manipulating stiffness is an important parameter of a manipulator, which can be used in the force or impedance control. A manipulator is open chain in mechanism generally, and the links always are rigid bodies, so the deformation on end-effector results from the joints mainly. The stiff mode can be written to the follow *equation. approximately*

$$M_i = k_i \Delta\theta_i \qquad i = 1, 2, \cdots, n \tag{1}$$

where M_i ——Torque of joint i

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$\Delta\theta_i$ ——Deformation of joint i

k_i ——Stiffness constant of joint i

If the gravitation and the friction in the joints are ignored, supposing there is a force vector $F \in \mathbf{R}^m$ on the manipulator's end effector, the equivalent joint torque can be written as

$$M = J^{\mathrm{T}} F \tag{2}$$

where $M \in \mathbf{R}^n$ ——Equivalent torque of the joints

$J \in \mathbf{R}^{m \times n}$ ——Jacobian matrix

It is well known that the deformations on the joints and the end-effector have a relationship as follows

$$\Delta x = J \Delta \theta \tag{3}$$

where Δx ——Micro displacement of the end-effector

$\Delta\theta$ ——Micro displacement of the joints

We write Eq.(1) as a matrix form and combine it with Eqs.(2) and (3), by some simple calculations, the relationship between Δx and F can be written as

$$\Delta x = \left(J k^{-1} J^{\mathrm{T}}\right) F \tag{4}$$

where

$$k = \begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & k_n \end{bmatrix} \tag{5}$$

If we define

$$C = J k^{-1} J^{\mathrm{T}} \tag{6}$$

Eq.(6) is to say the flexibility matrix of the end-effector. Whereas C^{-1} corresponds to the stiffness matrix in task space. The flexibility matrix C can be used to measure the static feature of Manipulator. Matrix C is also a function of the Jacobian, hence it is changeable in a large rang for it relations to the configuration and the construction parameters. The variable features of the manipulator in static can be used to complete some compliant and complex manipulation such as assemblage, polishing treatment and so on. Based on Eqs.(5) and (6), we can see matrix C is symmetric.

If we define

$$\mu = \sqrt{\det\left(C C^{\mathrm{T}}\right)} \tag{7}$$

and decompose matrix C by the singular value, from Eq.(7) we have

$$\mu = \prod_{i=1}^{m} \sigma_i \tag{8}$$

where σ_i, $i = 1, 2, \cdots, m$, denotes the singular value of matrix C. Therefore, the expression CC^{T} is a positive definite symmetric matrix, and an equation can be defined as

$$x^{\mathrm{T}}\left(C C^{\mathrm{T}}\right) x = 1 \tag{9}$$

Eq.(9) describes the equation of a generalized ellipsoid. This ellipsoid is to say the generalized flexibility ellipsoid (GFE). The principal axes of the ellipsoid are equal to the singular values of matrix C respectively. For some intuitionistic sake, a planar two links manipulator that the length of links $L_i = 1.0$ m, $i = 1, 2$, is regarded as an example (Fig.1), and some GFEs are shown in Fig.2 and Fig.3.

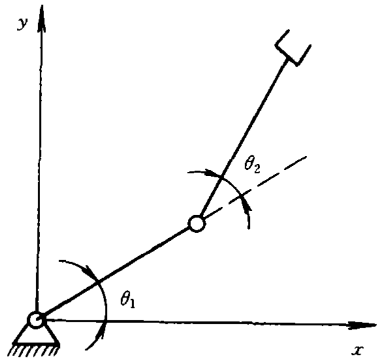

Fig.1 Planar 2R manipulator

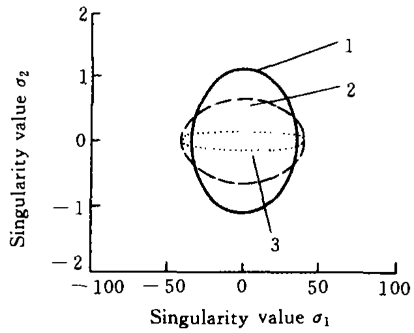

Fig.2 GFE of a full-actuated planar 2R manipulator

$L_1=L_2=1.0$ m

1. $\theta_1=\theta_2=60°$ 2. $\theta_1=\theta_2=35°$ 3. $\theta_1=\theta_2=20°$

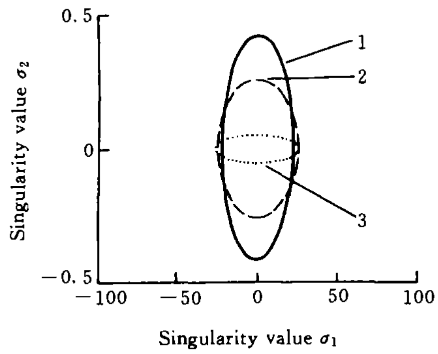

Fig.3 GFE of a full-actuated planar 2R manipulator

$L_1=0.5$ m, $L_2=1.0$ m

1. $\theta_1=\theta_2=60°$ 2. $\theta_1=\theta_2=35°$ 3. $\theta_1=\theta_2=20°$

These figures show that the measure depends on the configuration and the structure parameters. Whereas a full actuated manipulator is unable to be changed to the structure parameters generally. Therefore, the GFE can be changed by different configuration (Fig.2) but not the structure parameters (change from Fig.2 to Fig.3). When some passive joints are introduced into the full-actuated manipulator, for some convenience, supposing that the passive joints are equipped with brakes and position sensor so that the brakes can switch the passive joints between the free-swing mode and the locked mode, thus the underactuated redundant manipulator reveals some redundant DOFs in kinematics, but cannot be shown in "self-motion" for that the dimension of the inputs space is not more than that of the task space typically. On the other hand, switching the mode of the passive joins can reconfigure the underactuated manipulator, and the system reveals some dexterousness in adapting to different works.

2 FLEXIBILITY MATRIX

Supposed that there are s passive joints in an underactuated redundant manipulator, and the passive joints are equipped with brakes. When the passive joints are free-swing mode, the micro-motion equations of the manipulator can be written as

$$\Delta \boldsymbol{x} = \boldsymbol{J}_{\mathrm{a}} \Delta \boldsymbol{\theta}_{\mathrm{a}} + \boldsymbol{J}_{\mathrm{p}} \Delta \boldsymbol{\theta}_{\mathrm{p}} \tag{10}$$

where $\Delta \boldsymbol{x} \in \mathbf{R}^m$ ——Micro displacement of the end-effector

$\boldsymbol{J}_{\mathrm{a}} \in \mathbf{R}^{m \times n}$ ——Sub matrix of the Jacobian of the manipulator corresponding to the actuated joints

$\boldsymbol{J}_{\mathrm{p}} \in \mathbf{R}^{m \times s}$ ——Sub matrix of the Jacobian of the manipulator corresponding to the passive joints

$\Delta \boldsymbol{\theta}_{\mathrm{a}} \in \mathbf{R}^n$, $\Delta \boldsymbol{\theta}_{\mathrm{p}} \in \mathbf{R}^s$ ——Micro displacement in actuated joints and the passive joints respectively

When the passive joints are locked, the micro-motion equation can be described as

$$\Delta \boldsymbol{x} = \boldsymbol{J}_{\mathrm{l}} \Delta \boldsymbol{q} \tag{11}$$

where $\Delta \boldsymbol{x} \in \mathbf{R}^m$ ——Micro-displacement of the end-effector

$\boldsymbol{J}_{\mathrm{l}} \in \mathbf{R}^{m \times n}$ ——Jacobian of the manipulator as passive joints locked

$\Delta \boldsymbol{q} \in \mathbf{R}^n$ ——Micro-displacement in the actuated joints

It is obviously that Eqs.(11) and (3) have the same forms. Eqs.(10) and (11) indicate that an underactuated manipulator has two different model in kinematics. In other words, the system has the feature of multi-modes in kinematics. A planar 3R manipulator shown in Fig.4 can be regarded as an example of this. The second joint of the manipulator is passive, and the others are actuated. When the passive joint is free, $\boldsymbol{\theta} \in \mathbf{R}^3$ can be chosen as the generalized coordinates. If the passive joint is locked, the DOFs of the manipulator changes to two, and the generalized coordinates can be selected as $\boldsymbol{q} \in \mathbf{R}^2$. For the reason of $\boldsymbol{q} \neq \boldsymbol{\theta}$ obviously, thus the Jacobian matrix satisfies the relationship of $\boldsymbol{J}_{\mathrm{l}} \neq \begin{bmatrix} \boldsymbol{J}_{\mathrm{a}} & \boldsymbol{J}_{\mathrm{p}} \end{bmatrix}$.

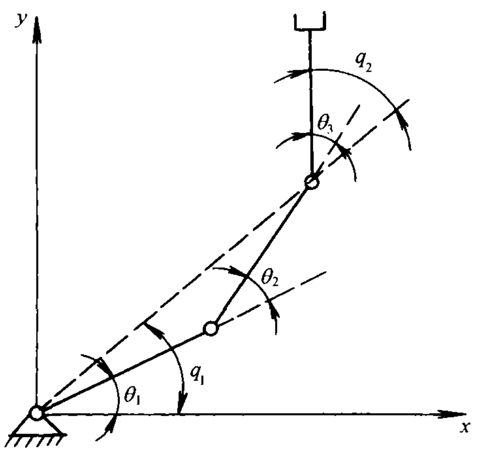

Fig.4 Planar 3R manipulator

By reason that the underactuated manipulator have different kinematics modes, one can select an optimal configuration and reconfigure the manipulator for adapting to a different task. An essential problem is how to predict the performance of the manipulator for full-actuated model based on the underactuated mode of it. Unlike a full-actuated redundant manipulator, the underactuated redundant manipulator cannot improve the performance by itself and implement a manipulation task simultaneously to the reason of fewer dimensions in input space than the task space. A feasible approach is to decompose these tasks to indifferent time for actualizing. For example, when the manipulator is working in the underactuated mode, one can reconfigure the mechanism for an appropriate configuration. Whereas when the manipulator is working in the full-actuated mode, one can control it to manipulation. Substantively, the manipulator working in underactuated mode can realize some manipulation such as position control [5] or discrete point-to-point motion [4]. But this is not the central of this paper. We pay attention to the static feature and self-reconfiguration control method of the underactuated redundant manipulator.

The kinematical equations of the two modes of the underactuated manipulator can be established by many methods (such as Denavit-Hartenberg method), but there is a difficulty in the structure parameter to be defined for a multi-DOF manipulator that is complicated in mechanism. To resolve this problem, next we analyze the relationship between the two modes of the underactuated redundant manipulator.

Given a special configuration of the manipulator, and supposing that has $n \geqslant m$, the deformations of the end effector under the two modes of the mechanism will be the same. This can be expressed as

$$\boldsymbol{J}_{\mathrm{l}} \Delta \boldsymbol{q} = \boldsymbol{J}_{\mathrm{a}} \Delta \boldsymbol{\theta}_{\mathrm{a}} + \boldsymbol{J}_{\mathrm{p}} \Delta \boldsymbol{\theta}_{\mathrm{p}} \tag{12}$$

Let

$$\boldsymbol{J}_{\mathrm{a}} \Delta \boldsymbol{\theta}_{\mathrm{a}} + \boldsymbol{J}_{\mathrm{p}} \Delta \boldsymbol{\theta}_{\mathrm{p}} = 0 \tag{13}$$

That means the micro-motion occurred in joint space does not change the position of the end effector. From Eq.(13), an expression can be written as

$$\Delta \boldsymbol{\theta}_{\mathrm{p}} = -\boldsymbol{J}_{\mathrm{p}}^{+} \boldsymbol{J}_{\mathrm{a}} \Delta \boldsymbol{\theta}_{\mathrm{a}} \tag{14}$$

where $(\bullet)^{+}$ denotes the Moore-Penrose pseudoinverse. Substituting Eq.(14) into Eq.(12), we obtain

$$\boldsymbol{J}_{\mathrm{l}} \Delta \boldsymbol{q} = \left[\left(\boldsymbol{I} - \boldsymbol{J}_{\mathrm{p}} \boldsymbol{J}_{\mathrm{p}}^{+} \right) \boldsymbol{J}_{\mathrm{a}} \right] \Delta \boldsymbol{\theta}_{\mathrm{a}} \tag{15}$$

Eq.(15) describes the same configuration of the two modes of the underactuated manipulator. Therefore the micro-motion denoted by the two kinds of generalized coordinates will be same. Let $\Delta \boldsymbol{q} = \Delta \boldsymbol{\theta}_{\mathrm{a}}$, a formulation is obtained as

$$\boldsymbol{J}_{\mathrm{l}} = \left[\left(\boldsymbol{I} - \boldsymbol{J}_{\mathrm{p}} \boldsymbol{J}_{\mathrm{p}}^{+} \right) \boldsymbol{J}_{\mathrm{a}} \right] \tag{16}$$

Eq.(16) shows the relationship of the Jacobian in the two modes, which can be used to predict the performance of the full-actuated mode. Substituting Eq.(16) into Eq.(5), a new flexibility matrix of underactuated manipulator in full-actuated mode can be written as

$$\boldsymbol{C} = \boldsymbol{J}_{\mathrm{l}} \boldsymbol{k}^{-1} \boldsymbol{J}_{\mathrm{l}}^{\mathrm{T}} \tag{17}$$

The GFE for the underactuated manipulator can also be defined according as Eq.(7). Eq.(17) indicates the static feature of the system after the mechanism reconfigured. Once more, we show it by the planar 3R manipulator (Fig.4). For a nonredundant manipulator, if we give a point in the task space, there is only one flexibility ellipsoid corresponding to it. By contrary, there are many ones corresponding to a point in task space for a redundant mechanism. Supposing the links length of the 3R manipulator are $L_1 = L_2 = 0.5$ m and $L_3 = 1.0$ m, the initial configuration is $\theta_1 = 60°$, $\theta_2 = -60°$, $\theta_3 = -30°$, some of the GFEs corresponding to the initial position of the end-effector are given in Fig.5.

It is obviously there are a lot of configurations in joint space corresponding to one state of the task space. These configurations are corresponding to different generalized flexibility ellipsoids respectively. Thus an underactuated redundant manipulator has the capability of reconfiguration. Generally, we expect the generalized flexibility ellipsoid has a similar performance in different direction of the principle axes. In other words, the ellipsoid is more similar to a ball. So the first configuration has the best performance among the three configurations that is given in Fig.5.

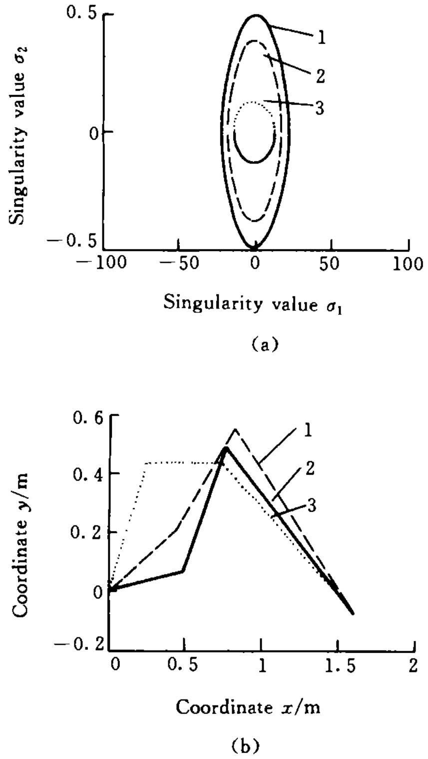

Fig.5 Generalized flexibility ellipsoids of 3R-underactuated manipulator

1. $\theta_1\approx 8.09°$, $\theta_2\approx 48.19°$, $\theta_3\approx -90.32°$
2. $\theta_1\approx 24.85°$, $\theta_2\approx 17.89°$, $\theta_3\approx -81.15°$
3. $\theta_1\approx 60°$, $\theta_2\approx -60°$, $\theta_3\approx -30°$

3 NONLINEAR CONTROL

For finding an approach that can control the underactuated manipulator efficaciously, we analyze the dynamic system. The dynamic equations for the underactuated manipulator can be written as

$$\boldsymbol{I}_{aa}\ddot{\boldsymbol{\theta}}_a + \boldsymbol{I}_{ap}\ddot{\boldsymbol{\theta}}_p + \boldsymbol{c}_a = \boldsymbol{M} \tag{18}$$

$$\boldsymbol{I}_{ap}^{T}\ddot{\boldsymbol{\theta}}_a + \boldsymbol{I}_{pp}\ddot{\boldsymbol{\theta}}_p + \boldsymbol{c}_p = \boldsymbol{0} \tag{19}$$

where $\boldsymbol{I} = \begin{bmatrix} \boldsymbol{I}_{aa} & \boldsymbol{I}_{ap} \\ \boldsymbol{I}_{ap}^{T} & \boldsymbol{I}_{pp} \end{bmatrix}$ denotes the mass inertia matrix, $\boldsymbol{c} = \begin{bmatrix} \boldsymbol{c}_a & \boldsymbol{c}_p \end{bmatrix}^{T}$ is the vector of Coriolis, centrifugal, gravitational and frictional torque, $\boldsymbol{M}$ is torque vector of the actuated joints, $\boldsymbol{\theta}_a$ is the generalized coordinates vector corresponding to the actuated joints, $\boldsymbol{\theta}_p$ whereas corresponding to the passive joints. Eq.(19) is second order nonholonomic constraints generally that have been proved by Jain, et al[6]. An underactuated redundant manipulator has the capability of improving the performance of the mechanism for a given position in task space by reconfiguration. For the reason of less dimension of input space than that of the joint space, the position control of the passive joint can only be realized by the dynamic coupling. Based on the Brockett 's theory[13], there is no smooth, static state feedback law that asymptotically stabilizes the system to a given configuration. Therefore, the results that have been proposed for controlling of the nonholonomic system are nonlinear, time varying, and discrete in nature. A nonlinear control method that has a manner of harmonic function for actuated joint is proposed in Ref.[17]. The basis of this method is that the passive joints is will deviate their equilibrium position when the actuated joint is moving in a periodic manner (Fig.6).

The harmonic motion is input to the actuated joint, such as

$$\boldsymbol{\theta}_a = A\cos\omega t \tag{20}$$

$$\dot{\boldsymbol{\theta}}_a = -A\omega\sin\omega t \tag{21}$$

$$\ddot{\boldsymbol{\theta}}_a = -A\omega^2\cos\omega t \tag{22}$$

where A ——Amplitude of the harmonic function
ω ——Angular frequency of the harmonic function

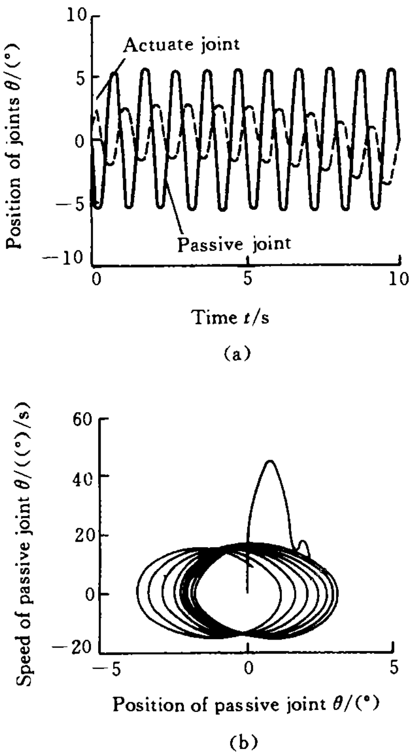

Fig.6 "Helicon motion" of two DOFs planar underactuated manipulator

If we approximate Eq.(22) by the first item of it's exponential progression, and substitute it to Eq.(19), we obtain

$$\ddot{\boldsymbol{\theta}}_p = -\boldsymbol{I}_{pp}^{-1}\left(\boldsymbol{c}_p - A\boldsymbol{I}_{ap}^{T}\omega^2\right) \tag{23}$$

Generally, the angular frequency ω is a large number, therefore the period $T = \dfrac{2\pi}{\omega}$ of the harmonic function is a small one, and the items such as $\boldsymbol{I}_{pp}^{-1}$, $\boldsymbol{c}_p$, and $\boldsymbol{I}_{ap}^{T}$ can be treated as constant in the time of a period T. The integral of Eq.(23) can be approximately written as

$$\boldsymbol{\theta}_p = \frac{1}{2}\boldsymbol{I}_{pp}^{-1}\left(\boldsymbol{c}_p - A\boldsymbol{I}_{ap}^{T}\omega^2\right)T^2 \tag{24}$$

Eq.(24) indicates an approximate deviation after a periodic time T. It is obviously that the value of the integral depends on the amplitude and angular frequency of the harmonic inputs. This is the reason of that the harmonic inputs in actuated joints can control the motion of the passive one.

4 SELF-RECONFIGURATION CONTROL

The self-reconfiguration needs a stable control technology. Based on the harmonic input nonlinear control method that given in section 3 briefly, next we will design a new control scheme to implement the self-reconfiguration motion. This method will be used to optimize the generalized flexibility ellipsoid for a given position in the task space.

Given $\boldsymbol{\theta}_d$ denotes an expected configuration that is derived from some optimizing method, $\boldsymbol{\theta}$ is the actual position of the manipulator.

Let

$$\boldsymbol{e} = \boldsymbol{\theta}_d - \boldsymbol{\theta} \tag{25}$$

where $\boldsymbol{e}$ ——Vector of joint position errors decomposing Eq.(24) to following form

$$\begin{bmatrix} \boldsymbol{e}_a \\ \boldsymbol{e}_p \end{bmatrix} = \begin{bmatrix} \boldsymbol{\theta}_{da} - \boldsymbol{\theta}_a \\ \boldsymbol{\theta}_{da} - \boldsymbol{\theta}_p \end{bmatrix} \tag{26}$$

a sliding model is given by

$$\boldsymbol{S}_a = \dot{\boldsymbol{e}}_a + k_1 \boldsymbol{e}_a \tag{27}$$

and the convergence law is selected as

$$\dot{\boldsymbol{S}}_a = -k_2 \operatorname{sgn}(\boldsymbol{S}_a) - k_3 \boldsymbol{S}_a \tag{28}$$

where, $k_1>0$, $k_2>0$, $k_3>0$, and sgn(•) indicates a sigmoid function, which has the form of

$$\operatorname{sgn}(s) = \begin{cases} 1 & s > 0 \\ -1 & s < 0 \end{cases}$$

If vector $\boldsymbol{S}_a$ has a manner of $\boldsymbol{S}_a = [s_{a1} \ \cdots \ s_{an}]^T$, following equation may be obtained

$$\dot{s}_{ai} s_{ai} < 0 \qquad i = 1, 2 \cdots, n \tag{29}$$

Eq.(27) manifests that the motion of the actuated joint will be stable for satisfying the Lyapunov stability theorem. Let the inputs of the actuated joints accord with Eqs.(20), (21), and select the parameters $k_d>0$, $k_d^2 - 4k_p > 0$, so that following relations can be obtained

$$\ddot{\boldsymbol{e}}_p + k_d \dot{\boldsymbol{e}}_p + k_p \boldsymbol{e}_p = 0 \tag{30}$$

Substituting the twice time derivation of the second row of Eq.(26) into Eq.(30), a result can be given by

$$\ddot{\boldsymbol{\theta}}_p = k_d \dot{\boldsymbol{e}}_p + k_p \boldsymbol{e}_p \tag{31}$$

Let the inputs of the actuated joints be

$$\ddot{\boldsymbol{\theta}}_a = -A\omega^2 \cos \omega t \tag{32}$$

substituting Eqs.(31) and (32) into Eq.(19), the following relations hold

$$\boldsymbol{I}_{ap}^T \left(-A\omega^2 \cos \omega t\right) + \boldsymbol{I}_{pp}\left(k_d \dot{\boldsymbol{e}}_p + k_p \boldsymbol{e}_p\right) + \boldsymbol{c}_p = \boldsymbol{0} \tag{33}$$

and the amplitude of the harmonic can be given by

$$A = (\boldsymbol{I}_{ap}^T \omega^2 \cos \omega t) + \left[\boldsymbol{I}_{pp}\left(k_d \dot{\boldsymbol{e}}_p + k_p \boldsymbol{e}_p\right) + \boldsymbol{c}_p\right] \tag{34}$$

Thus if the passive joint is not in the expected position, the control input of the actuated joint is defined by Eqs.(32) and (34). On the other hand, if the passive joint is in the expected position, the control input will change to the following equation. From Eq.(27), the time derivation is

$$\dot{\boldsymbol{S}}_a = \ddot{\boldsymbol{e}}_a + k_1 \dot{\boldsymbol{e}}_a = -\ddot{\boldsymbol{\theta}}_a + k_1 \dot{\boldsymbol{e}}_a \tag{35}$$

Combining Eqs.(28) and (35), the control law is given by

$$\ddot{\boldsymbol{\theta}}_a = k_1 \dot{\boldsymbol{e}}_a + k_2 \operatorname{sgn}(\boldsymbol{S}_a) + k_3 \boldsymbol{S}_a \tag{36}$$

It is obviously that the method suggested here is nonlinear and time varying, which obeys the Brockett's theory. Rearranging the algorithm above, the control law can be written as

$$\boldsymbol{S}_a = \dot{\boldsymbol{e}}_a + k_1 \boldsymbol{e}_a \tag{37}$$

When $\boldsymbol{e}_p = \boldsymbol{0}$ is satisfied

$$\ddot{\boldsymbol{\theta}}_a = k_1 \dot{\boldsymbol{e}}_a + k_2 \operatorname{sgn}(\boldsymbol{S}_a) + k_3 \boldsymbol{S}_a \tag{38}$$

When $\boldsymbol{e}_p \neq \boldsymbol{0}$ is satisfied

$$\ddot{\boldsymbol{\theta}}_a = \left[\boldsymbol{I}_{ap}^T \omega^2 \cos \omega t\right] + \left[\boldsymbol{I}_{pp}\left(k_d \dot{\boldsymbol{e}}_p + k_p \boldsymbol{e}_p\right) + \boldsymbol{c}_p\right] \omega^2 \cos \omega t \tag{39}$$

$$\boldsymbol{M} = \boldsymbol{I}_{aa} \ddot{\boldsymbol{\theta}}_a + \boldsymbol{I}_{ap} \ddot{\boldsymbol{\theta}}_p + \boldsymbol{c}_a \tag{40}$$

5 SIMULATION STUDY

In this section, the planar 3R manipulator is selected as a simulation model, which is shown in Fig.4. Supposing that the second joint of the manipulator is passive, and the other joints are actuated. If the initial configuration is $\theta_1 = 60°$, $\theta_2 = -60°$, $\theta_3 = -30°$, for improving the performance of the flexibility ellipsoid, a better configuration is $\theta_1 \approx 24.85°$, $\theta_1 \approx 17.89°$, $\theta_3 \approx -81.15°$, which is given in section 3. We regard the later one as the expected configuration. In accordance with the method that is suggested in section 4, the simulation result is shown in Fig.7. Fig.7a indicates

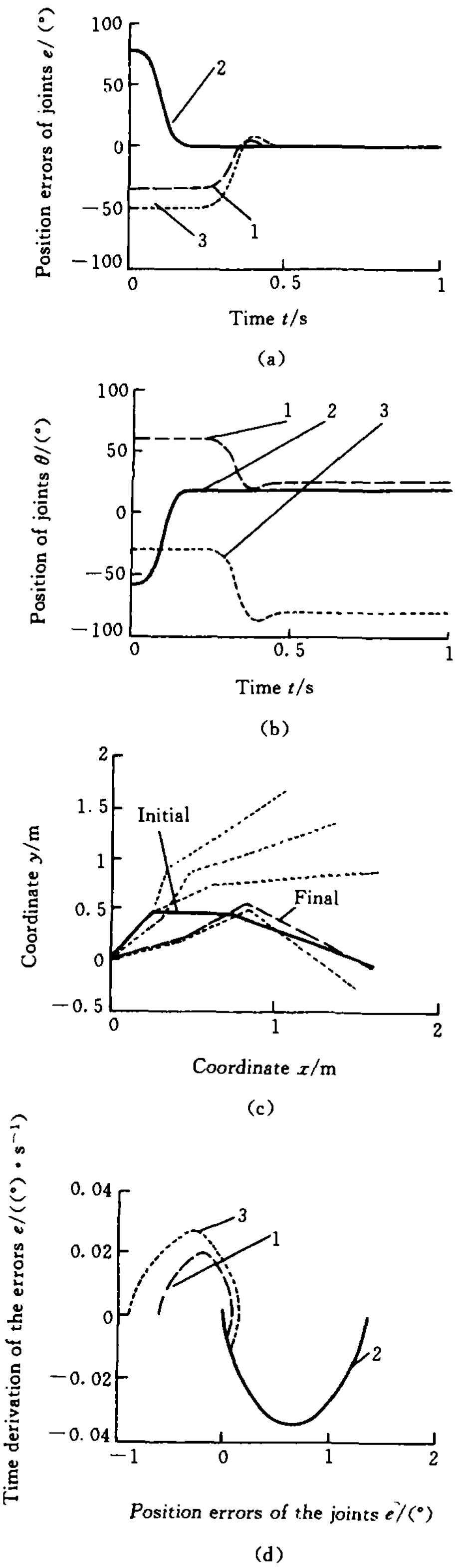

Fig.7 Self-reconfiguration of the 3R underactuated manipulator
1. Joint 1 2. Joint 2 3. Joint 3

the joints position errors with respect to time; Fig.7b is the joints trajectory respect to time; Fig.7c shows the transformation of the manipulator's configuration in the self-reconfiguring control; Fig.7d shows the phase relations between the speed and position of the joints. Obviously, the manipulator is reconfigured to the expected configuration by itself.

6 CONCLUSIONS

The underactuated technology is a crucial problem not only for fault tolerance of space robot systems but also for cooperation robot and metamorphic mechanisms. The underactuated redundant manipulator has the capability of realizing the mechanism reconfiguration by itself. The new generalized flexibility ellipsoid measure of the underactuated redundant manipulator with passive joints braked mode is suggested, and the measure can be used to optimize the static performance of the system. A novel nonlinear control algorithm based on the harmonic function can implement the motion of the self-reconfiguration. The simulation results by a three-DOFs underactuated manipulator prove the measure and the control algorithm is effectual.

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