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Optimisation of a CVT-Chain

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1. Abstract
In order to account for current and future requirements regarding the further improvement of the dynamical behaviour of a chain CVT (Continuous Variable Transmission) optimisation methods are substantial. The primary focus lies on the chain, which is the central part of the gear. In this work a concept to deal with this challenging task is introduced, leading to an optimisation tool for CVTs. First of all, a detailed dynamical model of the CVT is necessary to estimate the performance of the gear using numerical simulation techniques. The optimisation problem arising from the aim to optimise CVTs is identified in the next step. This includes the formulation of a target function quantifying the property which has to be optimised. For its evaluation simulated data is used. The optimisation problem is analysed in order to identify the requirements for a suitable optimisation algorithm. Implicit filtering will turn out to be the method of choice. For the implementation of complicated optimisation goals, a class of target functions is introduced. Therewith a tool for optimising CVTs is developed, which basically consists of the numerical simulation, the optimisation algorithm and the class of target functions. Its capability is shown by reducing the noise emission of a CVT which is achieved by obtaining optimal values for certain geometry parameters of the chain.

2. Keywords: CVT, optimisation, multibody dynamics, multibody simulation

3. Introduction
Continuously variable transmissions are a well-established alternative to common gears such as manual or conventional stepped automatic transmissions. They allow to drive in the most efficient operating range of the engine, which can enhance both performance and fuel economy.

As illustrated in Fig. 1, a continuous variable chain drive basically consists of the chain itself and

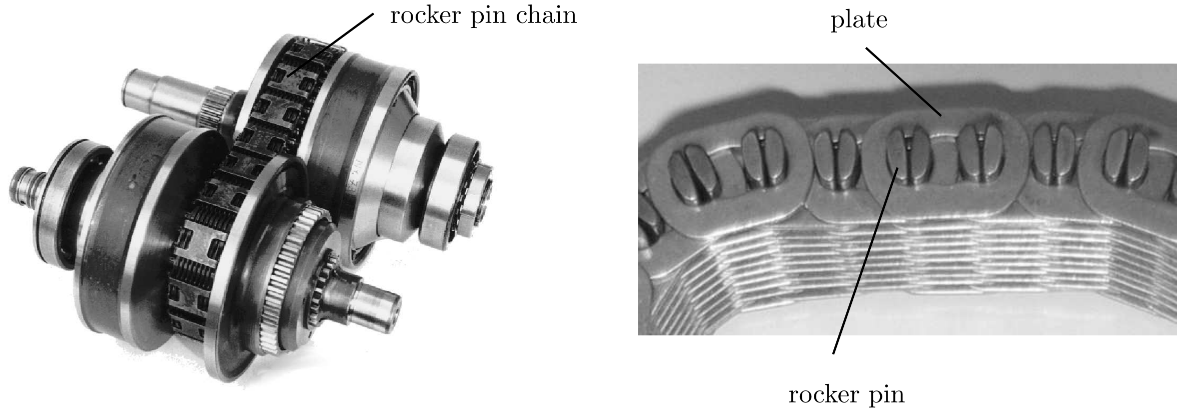

Figure 1: CVT-drive (left), rocker pin chain (right)

two pairs of cone discs, each with a fixed and an axial moveable sheave. The pulleys are coupled by the chain, which is composed of rocker pins connected by plates (Fig. 1). The power is transmitted by frictional forces between the discs and the ends of the rocker pins. Hydraulic actuators apply forces onto the moveable sheaves. Thus the radius of the chain in the pulleys and as a consequence the transmission ratio of the gear can be adjusted and changed continuously.

Nowadays CVTs have to satisfy high requirements to compete with common gears and therefore an improvement of certain dynamic properties of a CVT is substantial. An important aspect is a preferably small noise emission of the gear which can be achieved by a reduction of the force amplitudes in the bearings of the pulleys in a given frequency domain. Another goal is to minimise the maximum tensile forces in the chain links to increase the lifetime of the chain. Furthermore a higher efficiency of the gear is desired. These improvements are realised by calculating optimal parameters of the chain, which are for example the length and the stiffness of the links or the geometry of the rocker pin joints.

The model of the gear, which is used for the numerical simulation, is described in chapter 4. Next the optimisation problem is stated and analysed to find an appropriate algorithm for its solution. Section 6 covers a description of the optimisation process and its practical implementation. Finally, by solving the starting problem to minimise the noise emission of the CVT, the high potential of the introduced optimisation environment is disscussed.

4. Simulation Model of the CVT

A detailed dynamical model of the gear that covers all relevant effects of the CVT is the basis for the optimisation of the gear. The CVT is modelled as a multibody system which is shown in Fig. 2. It contains three essential components, the pulley sets, the chain and the contact model, describing the connection between chain and the pulleys. Each of the pulley sets has two degrees of freedom: one

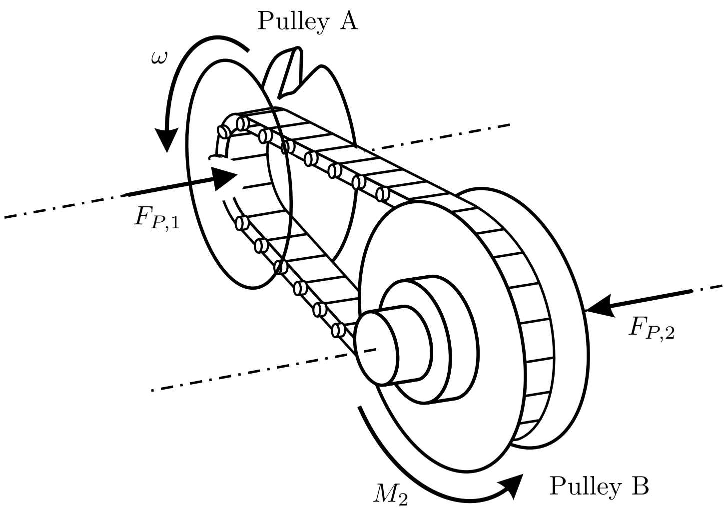

Figure 2: CVT-drive: Simulation model

rotational degree of freedom around the pulley axis and one translational degree of freedom of its movable sheave. Here the sheaves are assumed to be rigid, models which take into account elastic deformations are described in Sedlmayr [2]. Pulley A – the driving pulley – is kinematically excited by the angular velocity ω whereas an external torque M_2 acts on the driven pulley B. The transmission ratio is adjusted by the forces $F_{P,1}$ and $F_{P,2}$ which act on the movable sheaves.

In this work, a planar model of the chain is sufficient. In order to consider the discrete structure of the chain which causes the polygonal excitation, every single link is regarded. Each link consists of a massless spring damper element, representing the link plates and connecting two neighbouring joints (stiffness c, damping coefficient d). A mass m is located in each joint and is composed of the mass of the rocker pin pair as well as the half mass of each of the adjacent link plates.

The last component of the simulation model represents the frictional contact between the ends of the rocker pins and the sheaves. Neglecting its own dynamics, the pair of rocker pins can be modeled as one single, massless spring acting exclusively perpendicular to the model plane. Figure 3 shows the model of the bolt and the forces acting in the contact plane. For the derivation of the contact forces it is necessary to quantify the spring force F_B of the bolt. It depends on the length l_B and stiffness c_B as

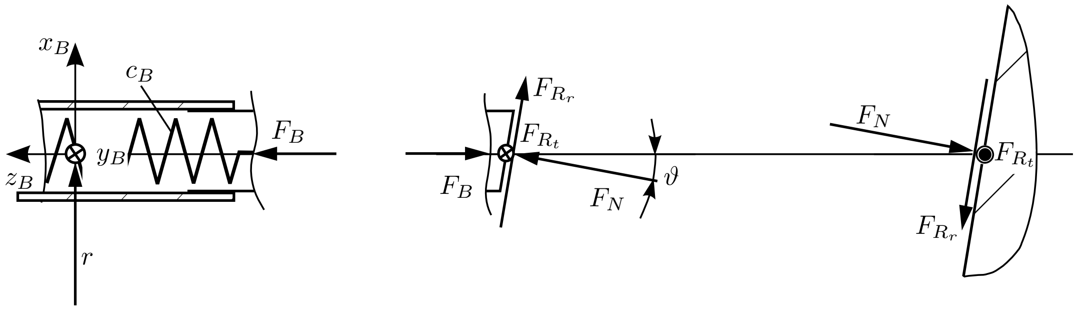

Figure 3: Model of a bolt

well as on the local distance of the surfaces s of the sheaves (Fig. 4):

$$F_B = \begin{cases} c_B\,(l_B - s\,) & \wedge \quad s \leq l_B \\ 0 & \wedge \quad s > l_B \end{cases} \quad . \tag{1}$$

The static equilibrium of forces perpendicular to the model plane provides a conditional equation for the normal force. Taking into account Eq. (1) it depends on the minimal coordinates of the according chain link and the contact force F_{R_r} in radial direction:

$$F_N = F_{R_r} \tan\vartheta + \frac{F_B}{\cos\vartheta} = F_{R_r} \tan\vartheta + \frac{c_B\,(l_B - s\,)}{\cos\vartheta} \tag{2}$$

To determine the remaining frictional forces as a function of the normal force, a continuous approximation of Coulomb's friction law (Fig. 4) is used where $\dot{\boldsymbol{g}}$ is the vector of the relative velocity in the contact:

$$\boldsymbol{F}_R = -\mu\, F_N\, \frac{\dot{\boldsymbol{g}}}{|\dot{\boldsymbol{g}}|}\,; \qquad \mu = \mu_0\,(1 - e^{-\frac{|\dot{\boldsymbol{g}}|}{\dot{g}_h}}\,) \tag{3}$$

A detailed exposition of the model can be found in Srnik [1] and Sedlmayr [2].

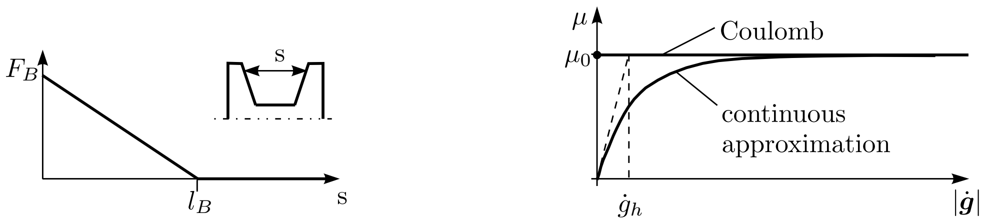

Figure 4: Bolt force and friction characteristic

Important for the further discussion of the optimisation task are the properties of the resulting equations of motion and the numerical simulation. As mentioned above, the model includes contacts with friction that may open and close during the simulation. This results in nonlinear equations of motion with a discontinuous right hand side in the form

$$\boldsymbol{M}\ddot{\boldsymbol{q}} = \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}, t). \tag{4}$$

Furthermore, the differential equations are stiff because of the stiffness of the springs representing the link plates. Both the discontinuities and the stiffness of Eq. (4) cause high simulation times.

5. Formulation and Analysis of the Optimisation Problem

In this section the *mathematical* optimisation problem, that arises when a CVT is to be optimised, is specified. Based on this mathematical formulation an optimisation algorithm is selected, which is capable to solve the problem efficiently. A goal function

$$f(\boldsymbol{p}) = f(\boldsymbol{q}(\boldsymbol{p}), \dot{\boldsymbol{q}}(\boldsymbol{p})) \tag{5}$$

has to be defined in dependence of given optimisation parameters $\boldsymbol{p}$, that quantifies the optimisation target. The complete solution set

$$(\boldsymbol{q}(\boldsymbol{p}), \dot{\boldsymbol{q}}(\boldsymbol{p})) := \{(\boldsymbol{q}(\boldsymbol{p}, t_i), \dot{\boldsymbol{q}}(\boldsymbol{p}, t_i)), i = 0, \ldots, k\} \tag{6}$$

of the initial value problem, which is based on the dynamical system (4), is needed for each evaluation of f. The data has to be computed on an equidistant grid, as some routines used for the evaluation of the target function require it, e.g. the fast fourier transformation. In order to provide more flexibility to choose optimisation goals, arbitrarily complex target functions f must be allowed for evaluating the simulation data. Operations like *min*, *max*, *least squares* or even a *FFT* (Fast Fourier Transformation) may be used, as well as combinations of them. The objective is to minimise the target function f by finding optimal parameters $\boldsymbol{p}$ subject to upper and lower bounds for each component of $\boldsymbol{p}$. This leads to a *constrained optimisation problem*

$$\min_{\boldsymbol{p} \in \mathcal{B}} f(\boldsymbol{q}(\boldsymbol{p}), \dot{\boldsymbol{q}}(\boldsymbol{p})), \qquad \mathcal{B} = \{\boldsymbol{p} \in \mathrm{I\!R}^m : l_i \leq p_i \leq u_i \quad \forall i = 1, \ldots, m\}\,. \tag{7}$$

When looking for a convenient optimisation algorithm, the actual problem has to be analysed under consideration of the following two aspects: firstly the properties of the objective function and secondly the constraints in the parameter space. For the problem at hand, the constraints are simple bound constraints, which can be handled with a gradient projection method, see Kelley [3]. Indeed the difficulties lie in the objective function. As a time consuming simulation has to be performed for each evaluation, the desired method has to rely on as few function evaluations as possible. Therefore genetic algorithms are out of question. Former investigations have shown, that even optimisation problems concerning rather simple mechanical systems can result in target functions that are nonsmooth, have many local minima and may depend on the optimisation parameters chaotically. A possible optimisation surface for a mechanical problem may be shaped like the one in Fig. 5.

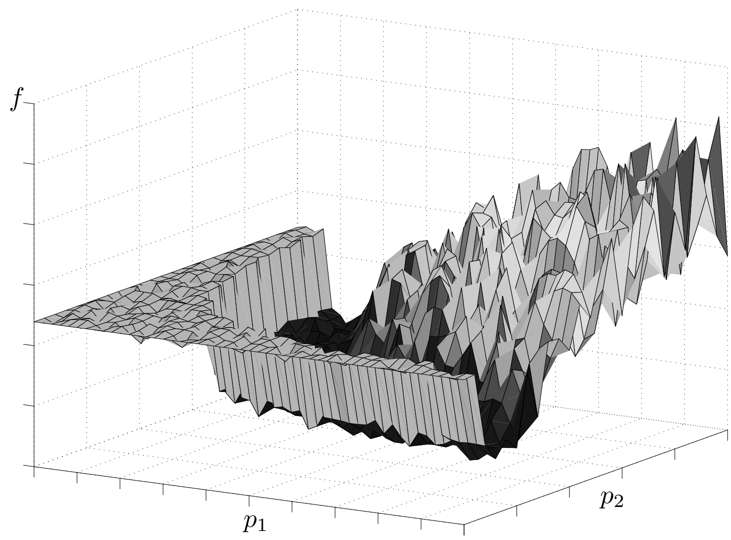

Figure 5: Possible optimisation surface

Such a worst case has to be expected. Another problem arises from the fact, that a quite large simulation model is implicitely integrated in the goal function f by the output of the numerical integration process. As stated before arbitray methods processing the simulation data have to be accepted. Hence it comes clear that it is not possible to calculate analytical gradients nor hessians. For such a problem the the optimisation algorithm *IFFCO* (Choi, et. al. [4]) seems to be very suitable. It is an implementation of the *implicit filtering* method for problems with bound contraints. The basic idea of *implicit filtering* is to utilise a sequence of difference increments for gradient approximation, starting with a rather big value which is reduced while the iteration proceeds, instead of using very small increments right at the beginning. This provides the opportunity not to get stuck in the first local minimum, but later when the increment gets small, a local minimum can be reached. A comprehensive description of implicit filtering is provided in Kelley [3]. An important feature of this implementation is, that the approximation of gradients by central differences is parallelised, what means a significant reduction of computation time for the whole optimisation process.

6. Optimisation Process

In this section the simulation model of the CVT and the optimisation algorithm are combined to form the desired optimisation tool for CVTs. Practical aspects for its implementation are to be discussed. First some additional components and features have to be provided. For a more comfortable implementation of complicated optimisation goals, a library of target functions is developed. It contains several target function prototypes (Tab. 1), which can be used directly or in combination to evaluate the objective function.

Table 1: Target function library.

Target functions
max
min
fast fourier transformation
least squares
...

The class of target functions computes the goal function value $f(\boldsymbol{p})$ directly from the simulation results and thus maintains an interface between the simulation and the optimisation algorithm. An extension of the simulation program is necessary, namely that the parameters $\boldsymbol{p}$, which describe the property that is optimised, can be changed from outside of the simulation. Further the functionality to return the required values, e.g. forces in a certain component, has to be established. The combination of the numerical simulation of the CVT, the optimisation algorithm and the library for target functions results in a powerful tool to optimise CVTs. Fig. 6 illustrates the interaction between the components of the tool. The master process is the optimisation algorithm. It calls the CVT simulation and passes

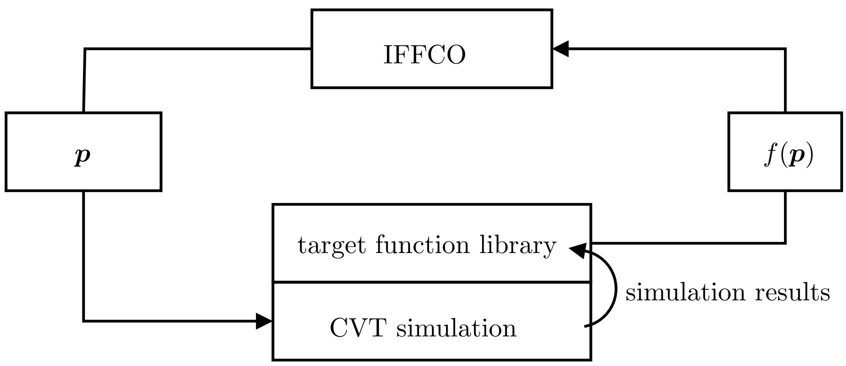

Figure 6: Interaction of the components of the optimisation tool

the parameter $\boldsymbol{p}$ whenever a function evaluation is needed. With this parameter set a complete simulation of the CVT is executed. The target function value, requested by the optimiser, is calculated from the simulation results by using the target function library. This process iterates until the optimisation algorithm terminates and an optimal parameter set $\boldsymbol{p}^*$ is found.

8. Optimisation Results

The high potential of the introduced tool for optimising CVTs is shown by solving an example problem. Noise emission is a significant problem of a CVT and it should be as small as possible. The noise is mainly generated by the polygonal effect, which occurs where the chain is entering the pulleys, due to the discrete structure of the chain. Approaches to reduce these effects are chains with links with different lengths or chains with an optimised rocker pin joint geometry. Here a chain with links having all the same length is investigated. An optimal link length shall be calculated and therefore the parameter vector is

$$\boldsymbol{q} = (a_{link}). \tag{8}$$

A measure for the accoustical performance of the gear are the amplitudes of the forces in the bearings of pulley B. The aim is to reduce the force peaks in the frequency domain, which refer to the polygonal frequency and its multiples. The following objective function is chosen

$$f = \sum_{\xi \in \{y,z\}} \int_{f_0}^{f_1} FFT(F_\xi) d\varsigma. \tag{9}$$

$F_\xi,\ \xi \in \{y, z\}$ refers to the two components of the support forces of the pulleys in the plane which is perpendicular to the axes of the pulleys. $FFT(s)$ is used as an abbreviation for the fast fourier transformation of the variable s. f_0 and f_1 are the lower and upper bound of the frequency domain in which the integral over the amplitudes of the forces shall be minimised. The target function f is evaluated numerically and includes two steps: firstly, the simulation of the entire CVT drive to get the forces F_ξ, which costs most of CPU-time and secondly, the calculation of the FFT and integration in Eq. (9), which is carried out automatically by the target function class.

The starting point for the presented optimisation is a PIV chain with the link length $a_{link} = 9.85$ mm. In the scope of this article only a special loadcase with the transmission ratio $i = 1.0$, the angular velocity $\omega =$ 104.7 rad/s of pulley A and the external torque $M_2 = -150$ Nm is considered. Using of the

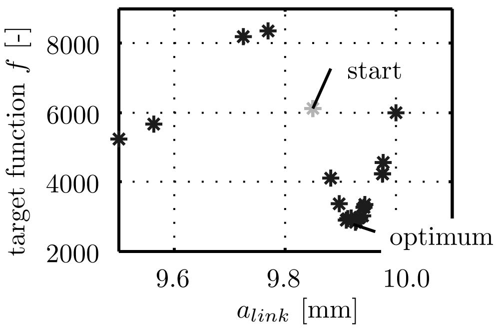

Figure 7: Influence of the link length a_{link} on the target function f.

introduced optimisation tool, the target function was reduced to 50%. Figure 7 shows the correlation between the link length a_{link} and the value of the target function f. It is obvious, that the starting configuration was not optimal with respect to the optimisation criterion at hand. The simulation results for the optimal link length $a_{link} = 9.93$ mm and for the special loading case as mentioned above are compared to the initial configuration in Fig. 8. Each major peak of the amplitudes of the two components of the support forces can be reduced to one third for that loadcase. This significant improvement of the acoustical performance of the gear is achieved by changing the link length by 0.08 mm.

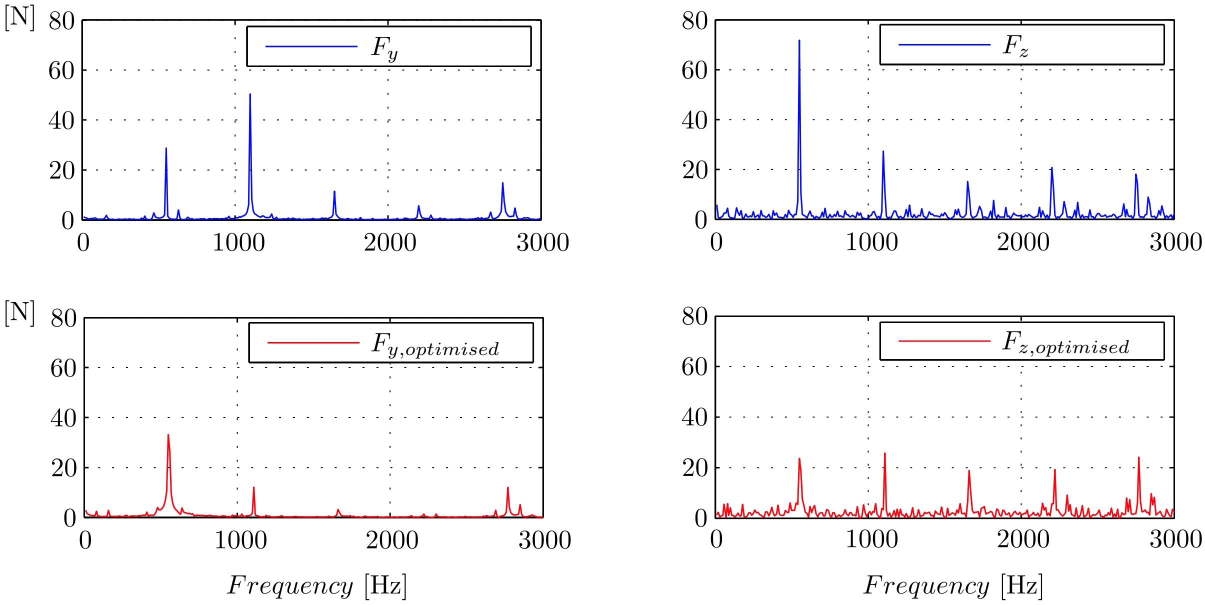

Figure 8: Comparison of the FFT of the components of support forces before and after optimisation: the major peaks of F_z at 550 Hz and of F_y at 1100 Hz are reduced to about one third by optimisation. ($i = 1.0$, $\omega =$ 104.7 rad/s, $M_2 = -150$ Nm).

9. Conclusions

A tool for optimising CVTs has been introduced. It includes the detailed simulation model of the CVT, an optimisation algorithm and a library of target functions. With the aim to find a suitable optimisation algorithm, the optimisation problem at hand is analysed and implicit filtering turns out to be a very good choice. The capability of the optimisation environment is illustrated by a practical example: the noise emission of the gear is reduced by an optimisation of the chain geometry. Complicated optimisation targets can be implemented quite comfortably and very good results are gained. The whole CVT will be optimised utilising this tool.

10. Acknowledgement

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11. References

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